CS448B :: 21 Oct 2009

Graph and Tree Layout

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Topics

Graph and Tree Visualization

- Tree Layout
- Graph Layout

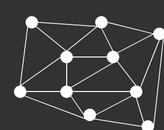
Goals

- Overview of layout approaches and their strengths and weaknesses
- Insight into implementation techniques

Graphs and Trees

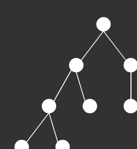
Graphs

- Model relations among data
- *Nodes* and *edges*



Trees

- Graphs with hierarchical structure
  - Connected graph with N-1 edges
- Nodes as *parents* and *children*



Spatial Layout

The primary concern of graph drawing is the spatial layout of nodes and edges

Often (but not always) the goal is to effectively depict the graph structure

- Connectivity, path-following
- Network distance
- Clustering
- Ordering (e.g., hierarchy level)

Applications of Tree / Graph Layout

Tournaments
Organization Charts
Genealogy
Diagramming (e.g., Visio)
Biological Interactions (Genes, Proteins)
Computer Networks
Social Networks
Simulation and Modeling
Integrated Circuit Design

Tree Visualization

Indentation
- Linear list, indentation encodes depth

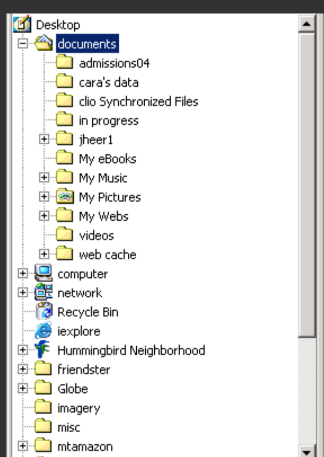
Node-Link diagrams
- Nodes connected by lines/curves

Enclosure diagrams
- Represent hierarchy by enclosure

Layering
- Layering and alignment

Tree layout is fast: O(n) or O(n log n), enabling real-time layout for interaction.

Indentation



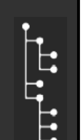
Places all items along vertically spaced rows

Indentation used to show parent/child relationships

Commonly used as a component in an interface

Breadth and depth contend for space

Often requires a great deal of scrolling



Node-Link Diagrams

Nodes are distributed in space, connected by straight or curved lines

Typical approach is to use 2D space to break apart breadth and depth

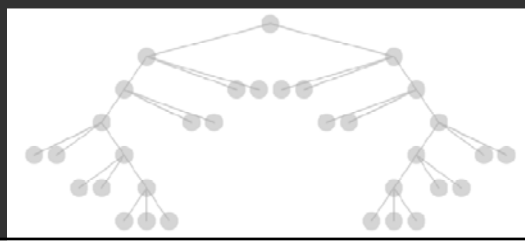
Often space is used to communicate hierarchical orientation (typically towards authority or generality)

Basic Recursive Approach

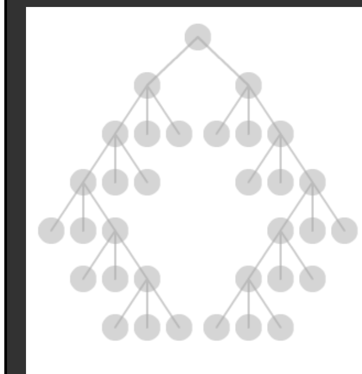
Repeatedly divide space for subtrees by leaf count

- Breadth of tree along one dimension
- Depth along the other dimension

Problem: exponential growth of breadth



Reingold & Tilford's Tidier Layout



Goal: make smarter use of space, maximize density and symmetry.

Originally for binary trees, extended by Walker to cover general case.

This extension was corrected by Buchheim et al to achieve a linear time algorithm.

Reingold-Tilford Layout

Design concerns

- Clearly encode depth level
- No edge crossings
- Isomorphic subtrees drawn identically
- Ordering and symmetry preserved
- *Compact layout (don't waste space)*

Reingold-Tilford Algorithm

Linear algorithm – starts with bottom-up pass of the tree

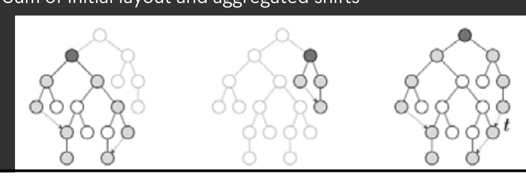
Y-coord by depth, arbitrary starting X-coord

Merge left and right subtrees

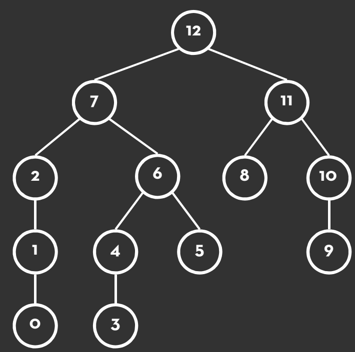
- Shift right as close as possible to left
  - Computed efficiently by maintaining subtree contours
- "Shifts" in position saved for each node as visited
- Parent nodes are centered above their children

Top-down pass for assignment of final positions

- Sum of initial layout and aggregated shifts



Reingold-Tilford Algorithm



Reingold-Tilford Algorithm

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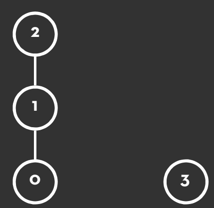

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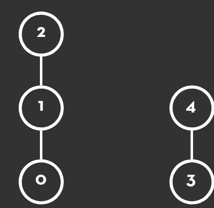

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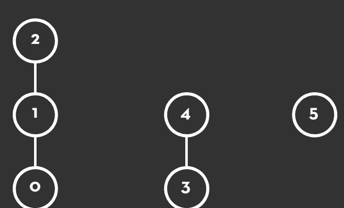

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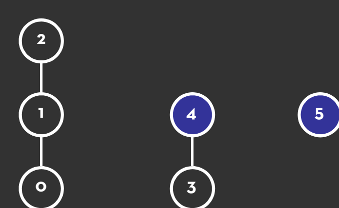

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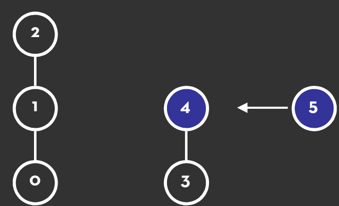

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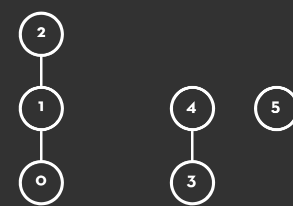

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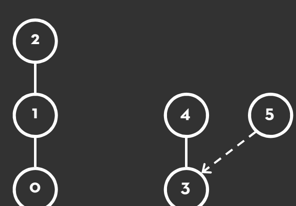

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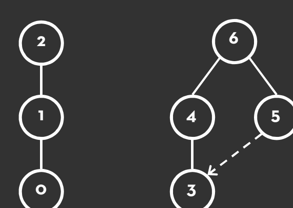

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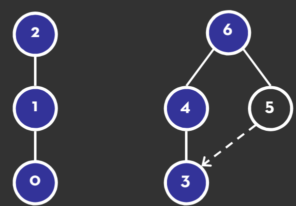

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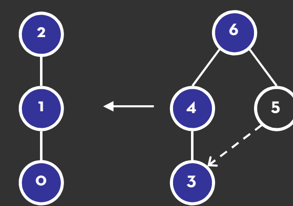

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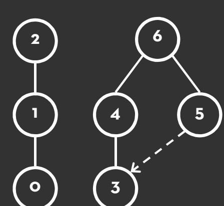

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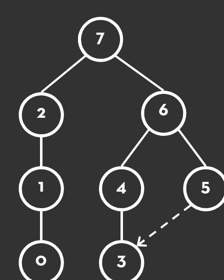

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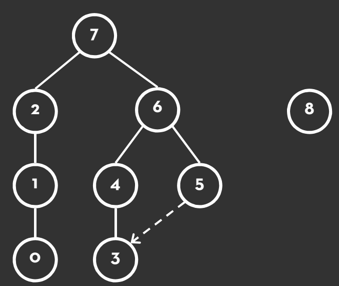

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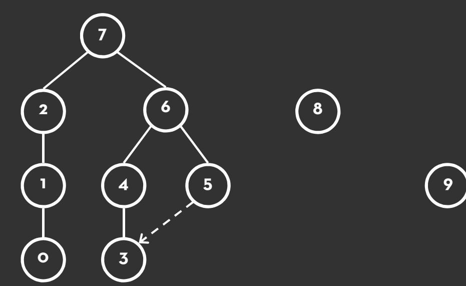

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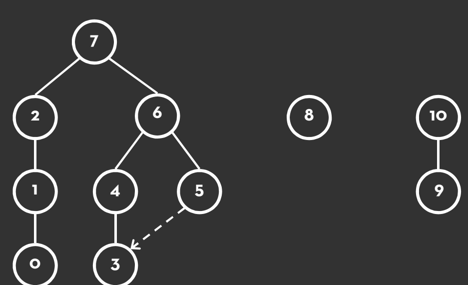

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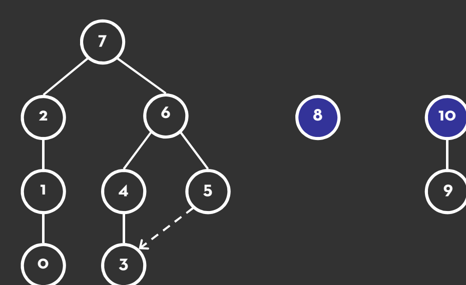

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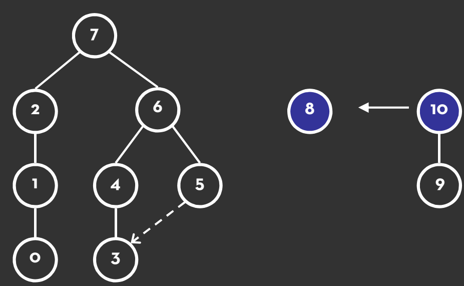

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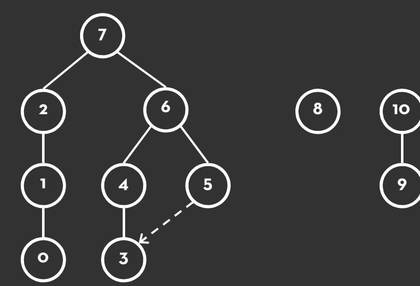

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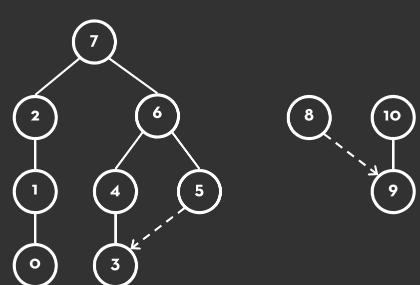

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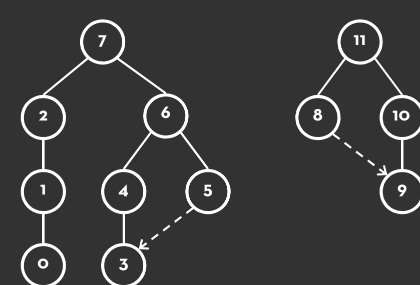

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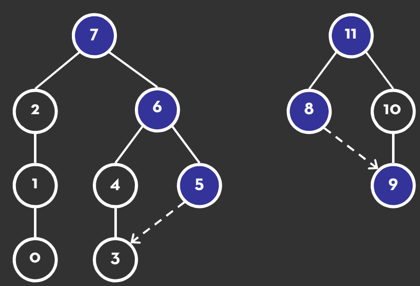

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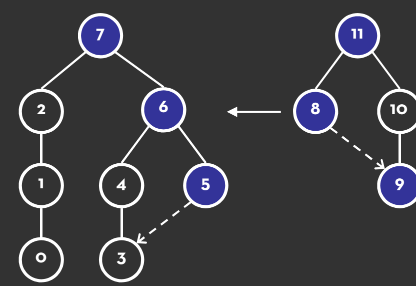

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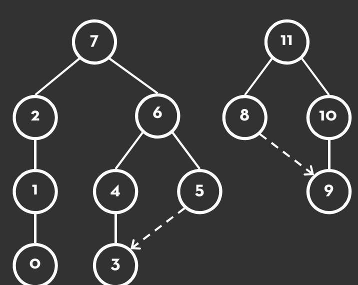

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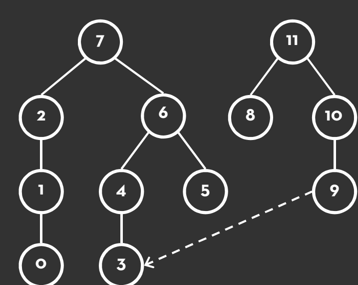

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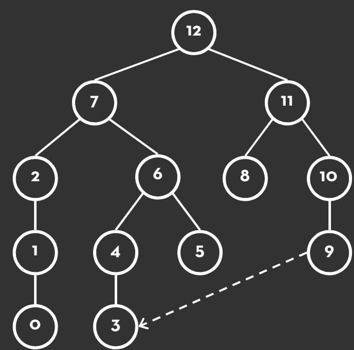
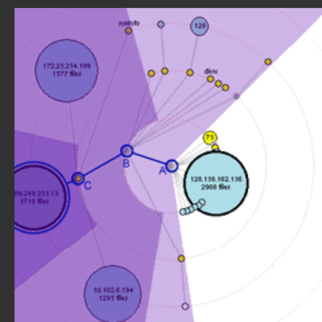


Radial Layout



Node-link diagram in polar co-ordinates.

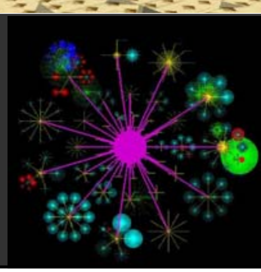
Radius encodes depth, with root in the center.

Angular sectors assigned to subtrees (typically uses recursive approach).

Reingold-Tilford approach can also be applied here.

Circular Drawing of Trees

Drawing in 3D to form Cone Trees



Balloon Trees can be described as a 2D variant of a Cone Tree. Not just a flattening process, as circles must not overlap.

Problems with Node-Link Diagrams

Scale

- Tree breadth often grows exponentially
- Even with tidier layout, quickly run out of space

Possible solutions

- Filtering
- Focus+Context
- Scrolling or Panning
- Zooming
- Aggregation

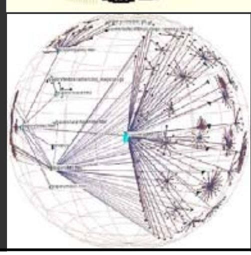
Visualizing Large Hierarchies

Indented Layout

Reingold-Tilford Layout

MC Escher, *Circle Limit IV*

Hyperbolic Layout

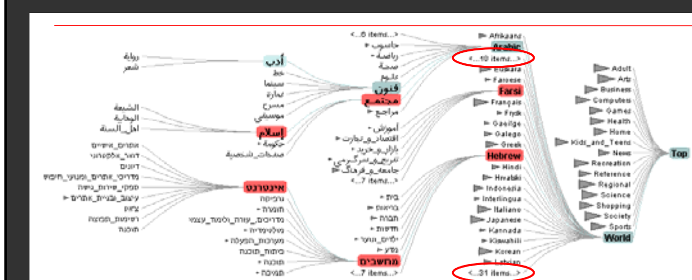


Perform tree layout in hyperbolic geometry, then project the result on to the Euclidean plane.

Why? Like tree breadth, the hyperbolic plane expands exponentially!

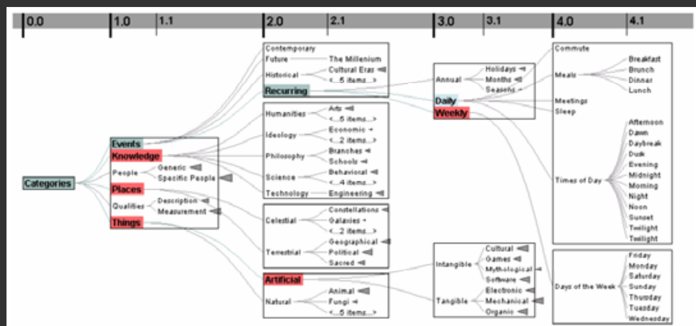
Also computable in 3D, projected into a sphere.

Degree-of-Interest Trees [AVI 04]



Space-constrained, multi-focal tree layout

Degree-of-Interest Trees



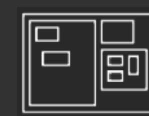
Cull "un-interesting" nodes on a per block basis until all blocks on a level fit within bounds.

Attempt to center child blocks beneath parents.

Enclosure Diagrams

Encode structure using spatial enclosure

Popularly known as TreeMaps



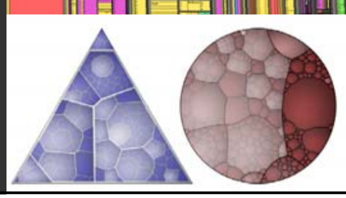
Benefits

- Provides a single view of an entire tree
- Easier to spot large/small nodes

Problems

- Difficult to accurately read depth

TreeMaps



Recursively fill space based on a size metric for nodes. Enclosure signifies hierarchy.

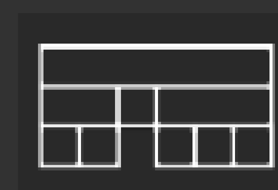
Additional measures can be taken to control aspect ratio of cells.

Often uses rectangles, but other shapes are possible, e.g., iterative Voronoi tesselation.

Layered Diagrams

Signify tree structure using

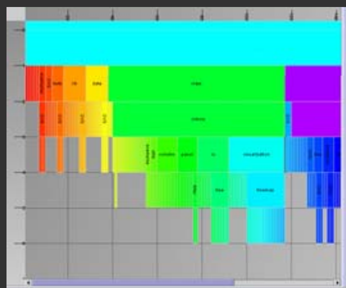
- Layering
- Adjacency
- Alignment

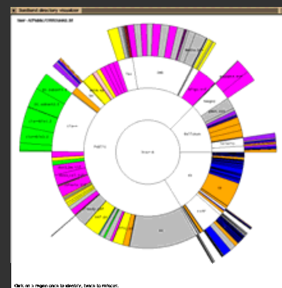


Involves recursive sub-division of space

We can apply the same set of approaches as in node-link layout.

Icicle and Sunburst Trees

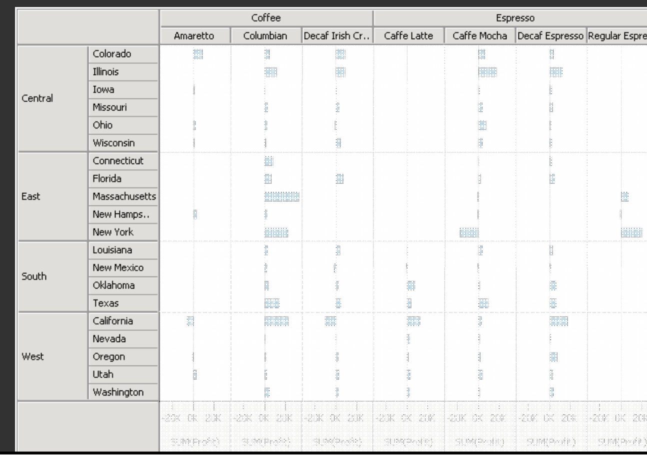




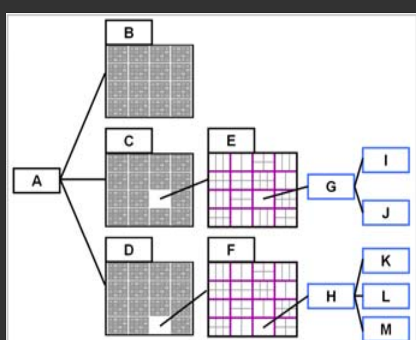
Higher-level nodes get a larger layer area, whether that is horizontal or angular extent.

Child levels are layered, constrained to parent's extent

Layered Tree Drawing



Hybrids are also possible…



"Elastic Hierarchies"

Node-link diagram with treemap nodes.

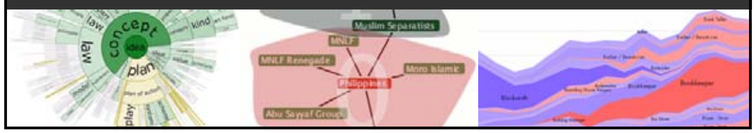
Administrivia

Assign 3: Interactive Visualization

Create an interactive visualization application. Choose a data domain and select an appropriate visualization technique.

1. Choose a data set and storyboard your interface
2. Implement the interface using tools of your choice
3. Submit your application and produce a final write-up

You may work individually or in groups of 2.
Due by *end of day* on **Wednesday, October 28**



Final Project

Design a new visualization technique or system
- Implementation of new design or system
- 8-10 page paper in conference paper format
- 2 Project Presentations

Schedule
- Project Proposal: **Wednesday, November 4** *(end of day)*
- Initial Presentation: **Monday, November 9 & Wednesday, November 11**
- Poster Presentation: **Wednesday, December 2** (tentative)
- Final Papers: **Monday, December 7** (by noon)

Logistics
- Groups of up to 3 people, graded individually
- Clearly report responsibilities of each member

Graph Visualization

Approaches to Graph Drawing

Direct Calculation using Graph Structure
- Tree layout on spanning tree
- Hierarchical layout
- Adjacency matrix layout

Optimization-based Layout
- Constraint satisfaction
- Force-directed layout

Attribute-Driven Layout
- Layout using data attributes, not linkage

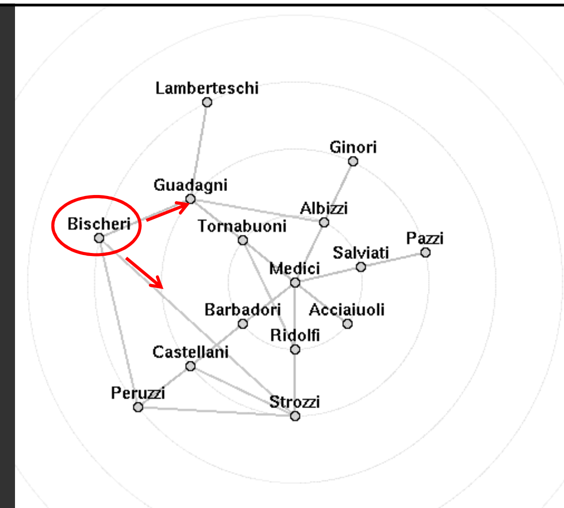
Spanning Tree Layout

Many graphs are tree-like or have useful spanning trees
- Websites, Social Networks

Use tree layout on spanning tree of graph
- Trees created by BFS / DFS
- Min/max spanning trees

Fast tree layouts allow graph layouts to be recalculated at interactive rates

Heuristics may further improve layout

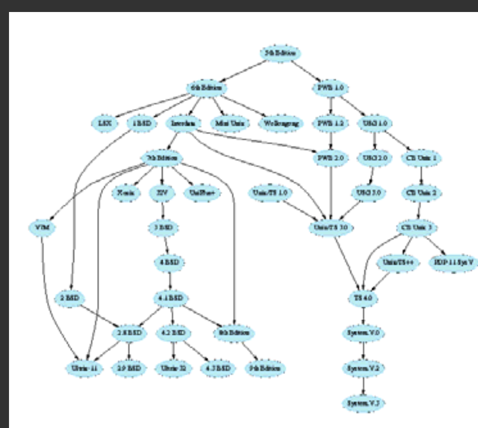


Spanning tree layout may result in arbitrary parent node

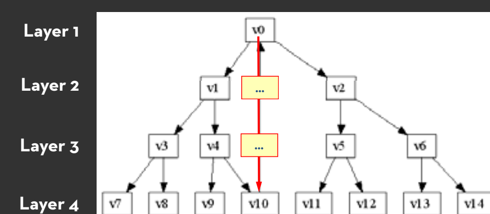
Sugiyama-style graph layout

Evolution of the UNIX operating system

Hierarchical layering based on descent



Sugiyama-style graph layout

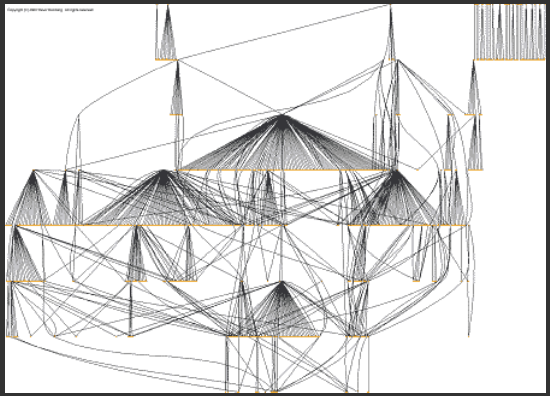


Assign nodes to hierarchy layers
- Reverse edges to remove cycles
- Create dummy nodes to “fill in” missing layers

Arrange nodes within layer, minimize edge crossings

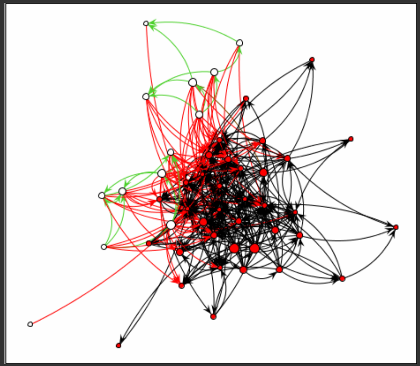
Route edges – layout splines if needed

Hierarchical graph layout



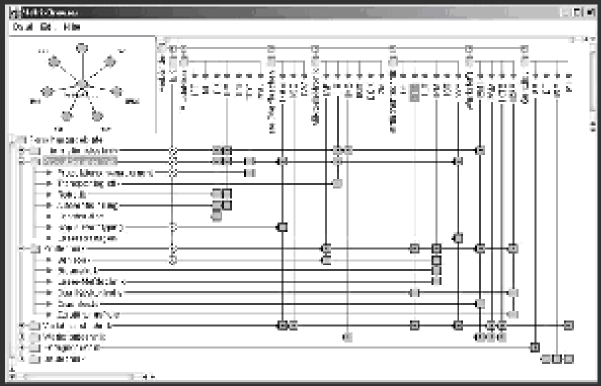
Gnutella network

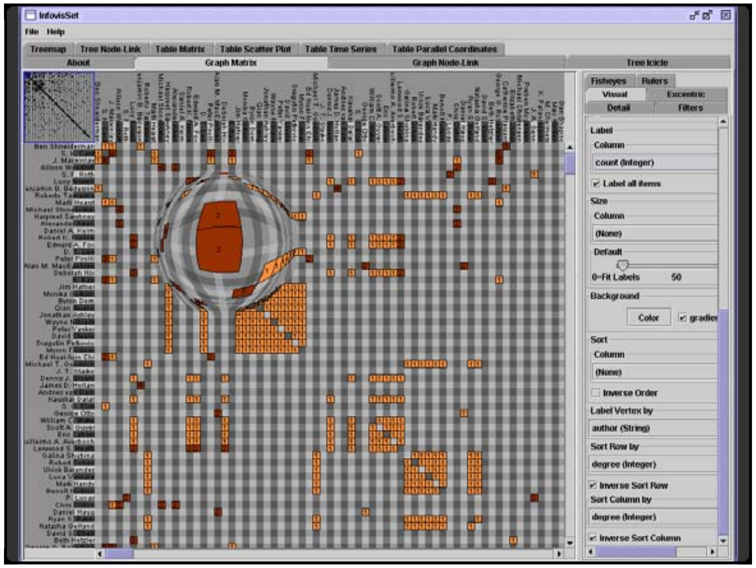
Limitations of Node-Link Layout

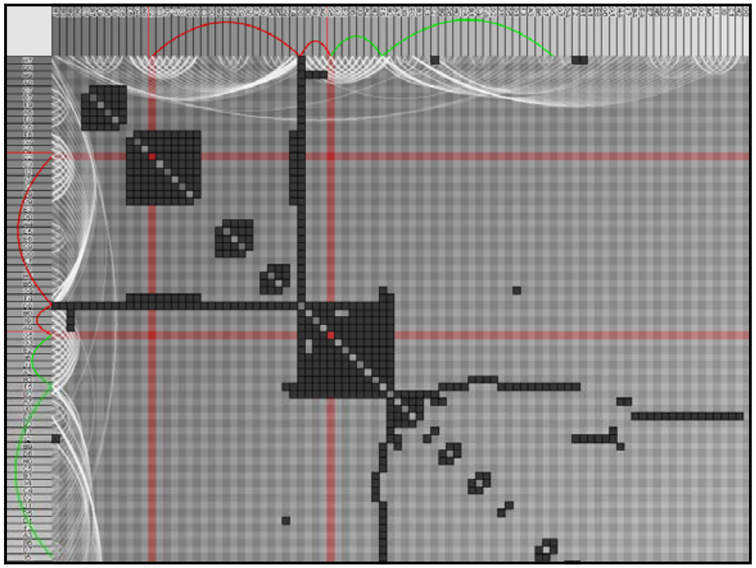


Edge-crossings and occlusion

Adjacency Matrices







Optimization Techniques

Treat layout as an *optimization problem*

- Define layout using a set of *constraints*: equations the layout should try to obey
- Use optimization algorithms to solve

Common approach for undirected graphs

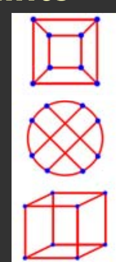
- *Force-Directed Layout* most common

We can introduce directional constraints

- *DiG-CoLa* (Di-Graph Constrained Optimization Layout) [Dwyer 05]

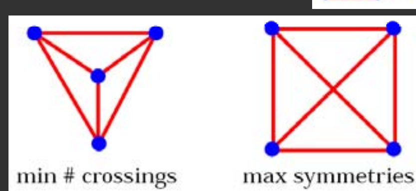
Optimizing "Aesthetic" Constraints

Minimize edge crossings
Minimize area
Minimize line bends
Minimize line slopes
Maximize smallest angle between edges
Maximize symmetry



but, can't do it all.

Optimizing these criteria is often NP-Hard, requiring approximations.

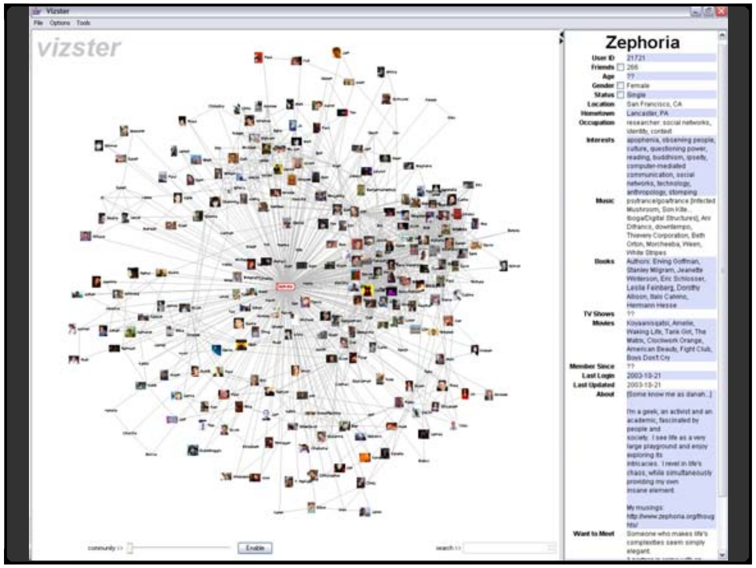


Force-Directed Layout

Edges = springs $F = -k * (x – L)$
Nodes = charged particles $F = G*m_1*m_2 / x^2$

Repeatedly calculate forces, update node positions

- Naïve approach $O(N^2)$
- Speed up to O(N log N) using quadtree or k-d tree
- Numerical integration of forces at each time step



Constrained Optimization Layout

Minimize stress function

$$\mathrm{stress}(X) = \Sigma_{i<j}\, w_{ij}\, (\, \|X_i - X_j\| - d_{ij}\,)^2$$

- X: node positions, d: optimal edge length,
- w: normalization constants
- Use global (*majorization*) or localized (*gradient descent*) optimization

→ Says: Try to place nodes d_{ij} apart

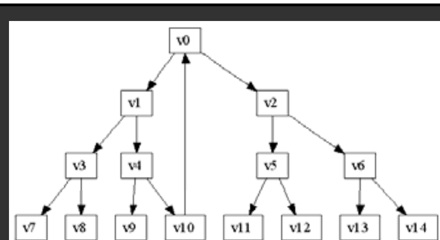
Add hierarchy ordering constraints

$$E_H(y) = \Sigma_{(i,j)\in E}\, (\, y_i - y_j - \delta_{ij}\,)^2$$

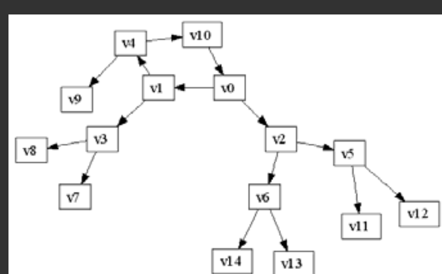
- y: node y-coordinates
- δ : edge direction (e.g., 1 for i→j, 0 for undirected)

→ Says: If *i* points to *j*, it should have a lower y-value

Sugiyama layout (dot)
Preserve tree structure

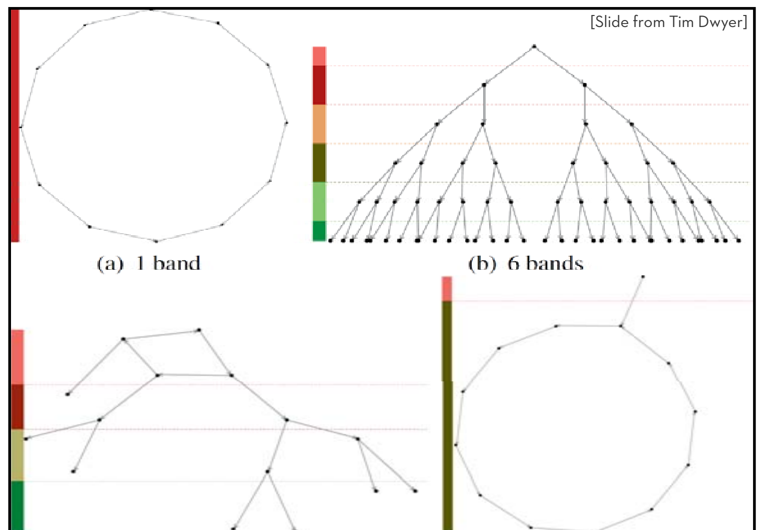


DiG-CoLa method
Preserve edge lengths



[Slide from Tim Dwyer]

[Slide from Tim Dwyer]



(a) 1 band (b) 6 bands (c) 4 bands (d) 2 bands

Attribute-Driven Layout

Large node-link diagrams get messy!

Is there additional structure we can exploit?

Idea: Use data attributes to perform layout

- e.g., scatter plot based on node values

Dynamic queries and/or brushing can be used to explore connectivity

Attribute-Driven Layout

The "Skitter" Layout

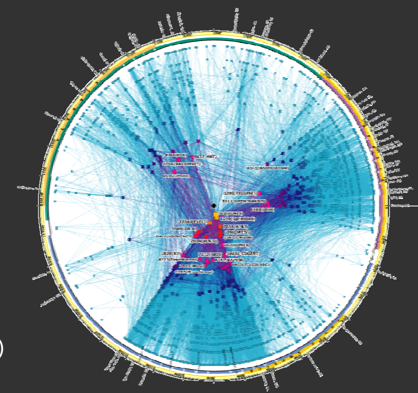
- Internet Connectivity
- Radial Scatterplot

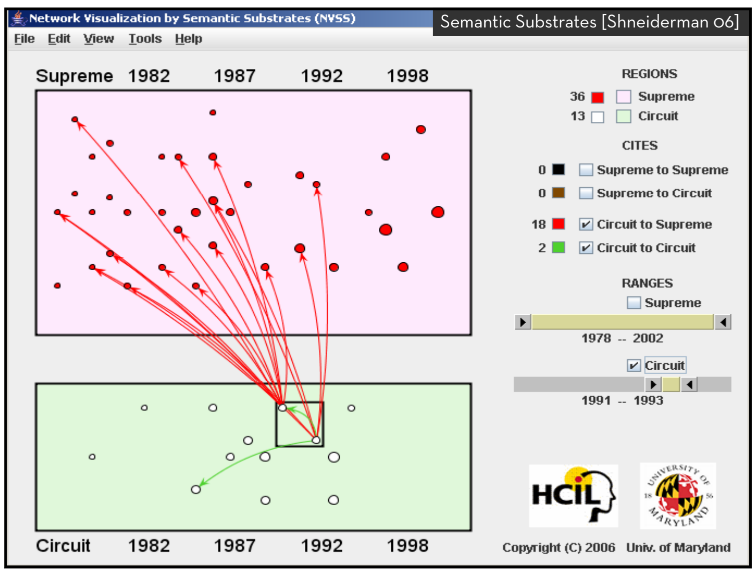
Angle = Longitude

- Geography

Radius = Degree

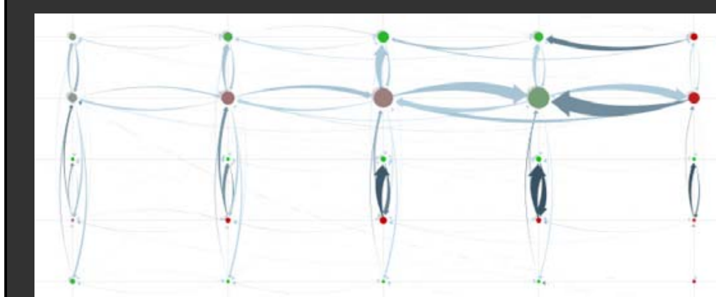
- # of connections
- (a statistic of the nodes)





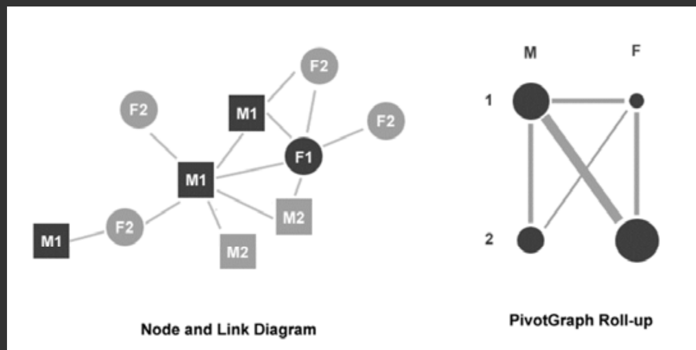
Semantic Substrates [Shneiderman 06]

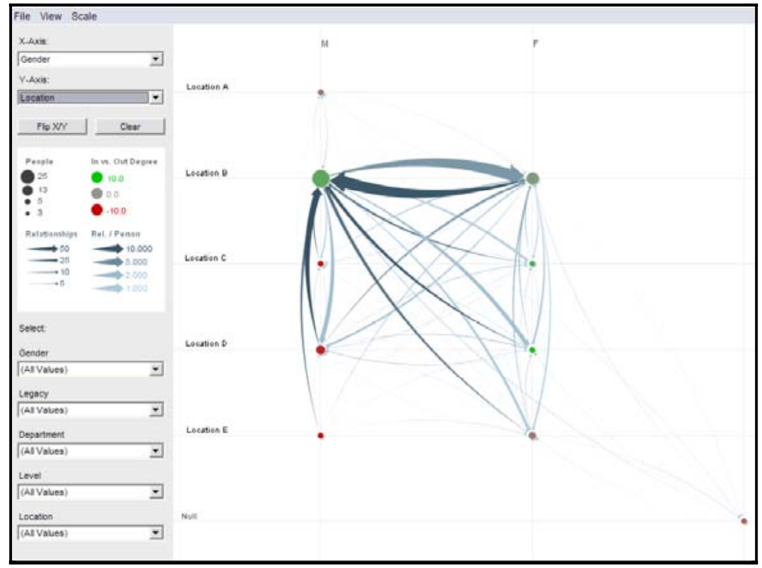
PivotGraph [Wattenberg 2006]



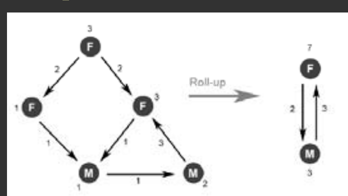
Layout aggregated graphs according to node attributes.
Analogous to pivot tables and trellis display.

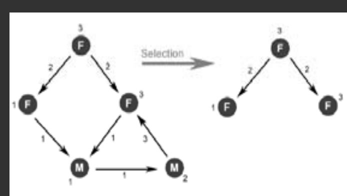
PivotGraph





Operators



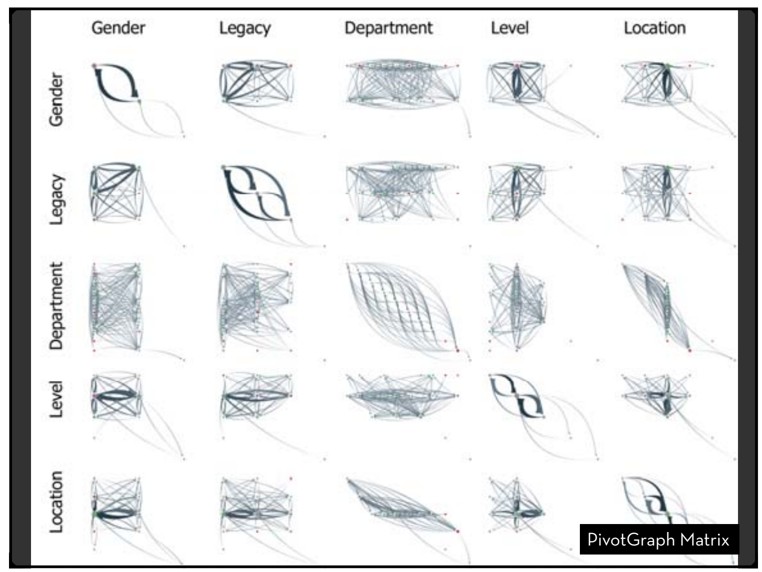


Roll-Up

Aggregate items with matching data values

Selection

Filter on data values



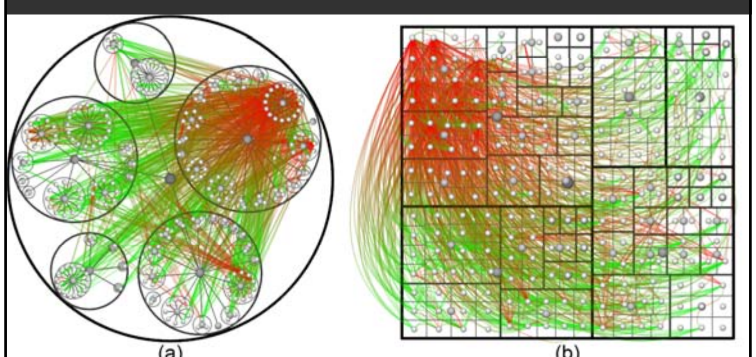
PivotGraph Matrix

Limitations of PivotGraph

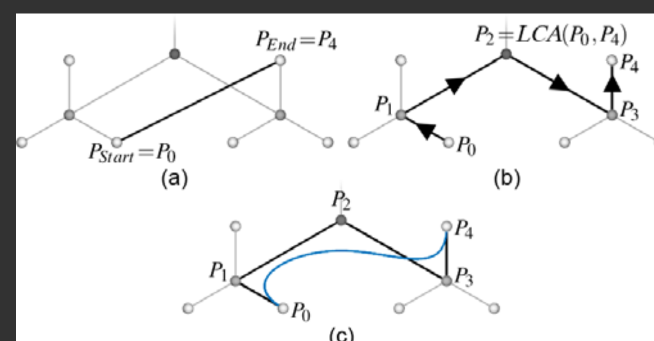
Only 2 variables (no nesting as in Tableau)
Doesn't support continuous variables
Multivariate edges?

Hierarchical Edge Bundles

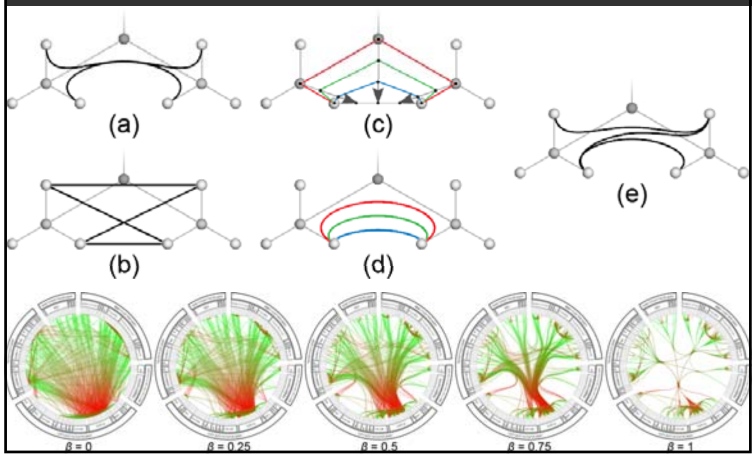
Trees with Adjacency Relations

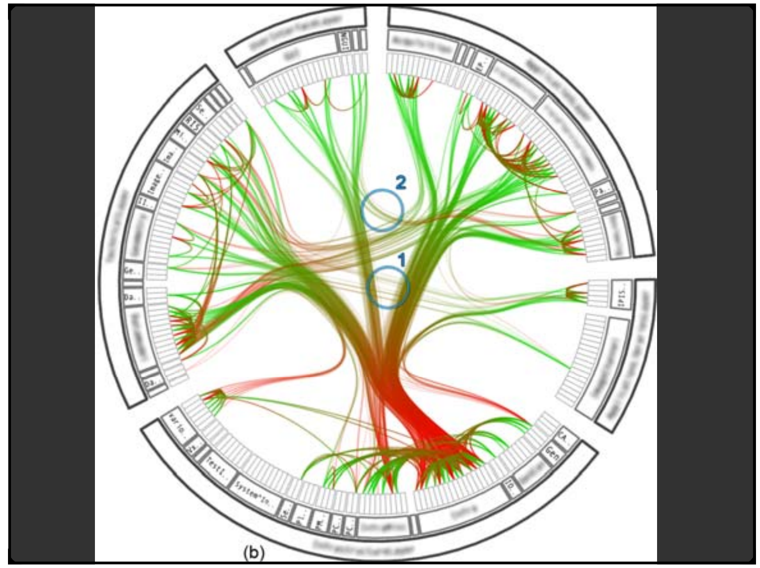


Bundle Edges along Hierarchy



Configuring Edge Tension





Summary

Tree Layout

- Indented / Node-Link / Enclosure / Layers
- How to address issues of scale?
  - Filtering and Focus + Context techniques

Graph Layout

- Tree layout over spanning tree
- Hierarchical "Sugiyama" Layout
- Optimization Techniques
- Attribute-Driven Layout