

# Disentangled Fair Representations

May 29, 2020

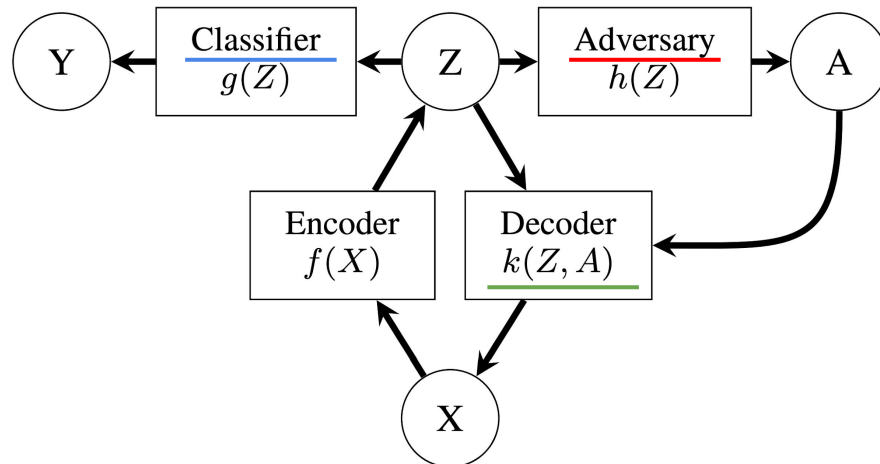
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CS 335: Fair, Accountable, and Transparent (FAcCT) Deep Learning  
Stanford University

# Fairness Through Adversarial Learning

- Adversarial Learning

$$L(f, g, h, k) = \alpha L_C(g(f(X, A)), Y) + \beta L_{Dec}(k(f(X, A), A), X) + \gamma L_{Adv}(h(f(X, A)), A)$$



$$\underset{f, g, k}{\text{minimize}} \underset{h}{\text{maximize}} \mathbb{E}_{X, Y, A} [L(f, g, h, k)]$$

[Madras et al, 2018](#)

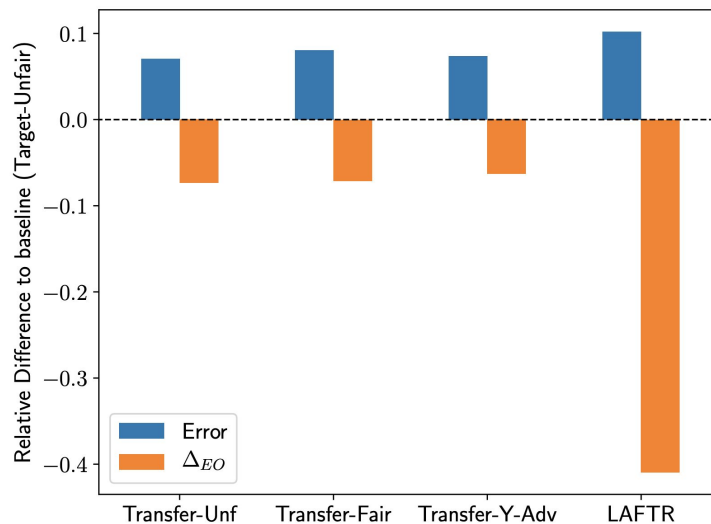
# Transfer Fair Representations

- Heritage Health Dataset
  - Comprises insurance claims and physician records
  - Task 1 - Predict Charlson index (prediction of 10 year survival of patients) trained using equalized odds adversarial objective
  - Task 2 - Same input, task becomes predicting a patient's insurance claim corresponding to a specific medical condition

Transfer- unf - MLP with no fairness constraints

Transfer- fair - MLP with fairness constraints in [Bechavod et al, 2017](#)

Transfer - Y - Adv baseline in [Zhang et al, 2018](#)

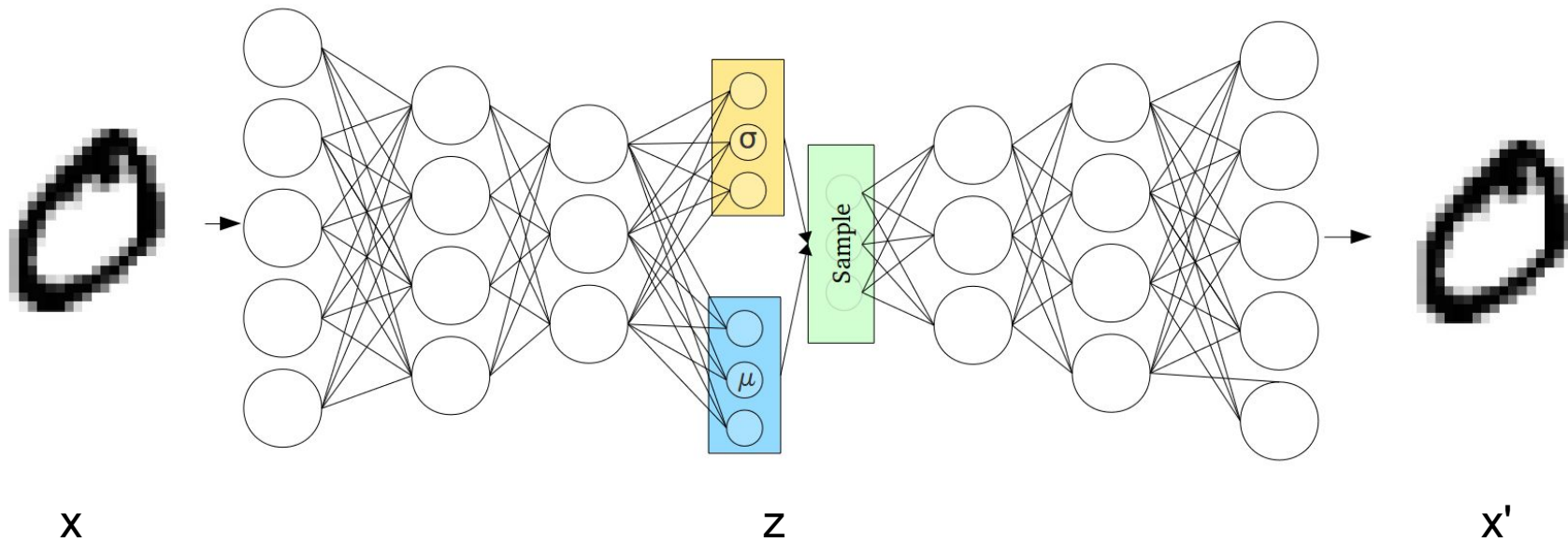


[Madras et al, 2018](#)

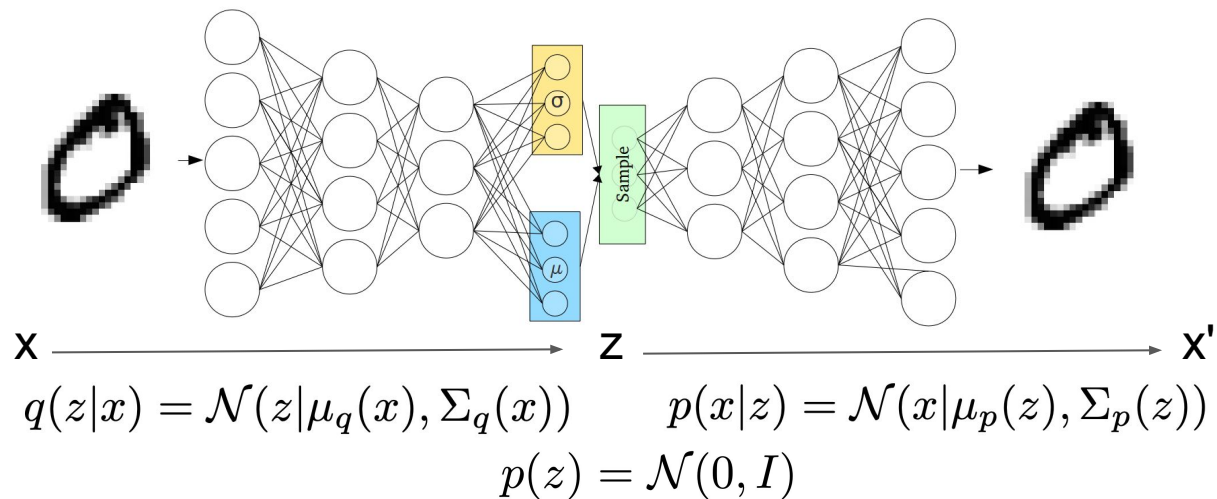
# Outline

- Disentangled Representations
- Flexibly Fair Representation
- Orthogonal Disentangled Fair Representations
- Measurements for Disentangled Fair Representations

# VAE Revisited



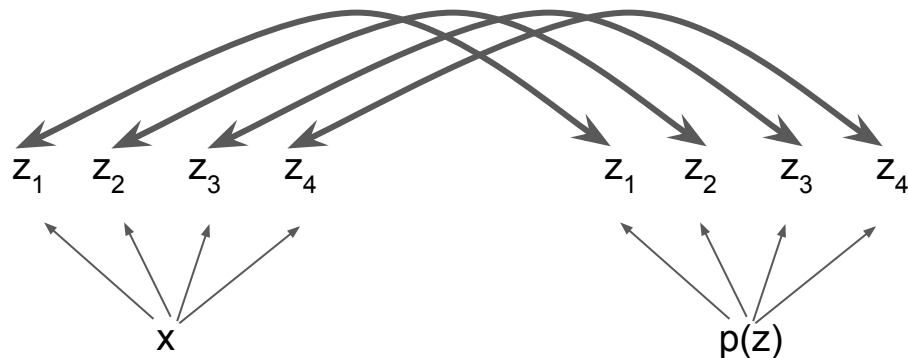
# VAE Revisited



$$L_{\text{VAE}}(p, q) = \underbrace{\mathbb{E}_{q(z|x)}}_{\text{encoder}} \underbrace{[\log p(x|z)]}_{\text{decoder}} - \underbrace{D_{KL} [q(z|x) || p(z)]}_{\text{KL divergence}}$$

# Disentanglement in VAE

$$L_{\text{VAE}}(p, q) = \mathbb{E}_{q(z|x)} [\log p(x|z)] - \underbrace{D_{KL} [q(z|x) || p(z)]}$$

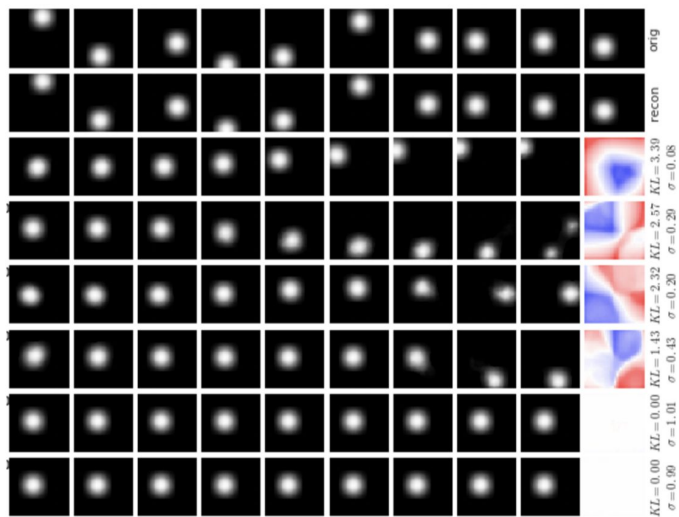


$$p(z) = \prod_j p(z_j)$$

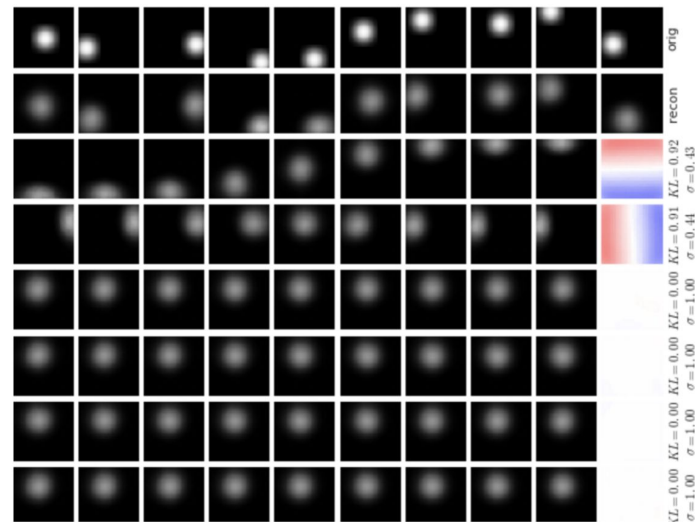
# $\beta$ -VAE

$$L_{\beta\text{VAE}}(p, q) = \mathbb{E}_{q(z|x)} [\log p(x|z)] - \beta D_{KL} [q(z|x) || p(z)]$$

$\beta = 1$



$\beta = 150$





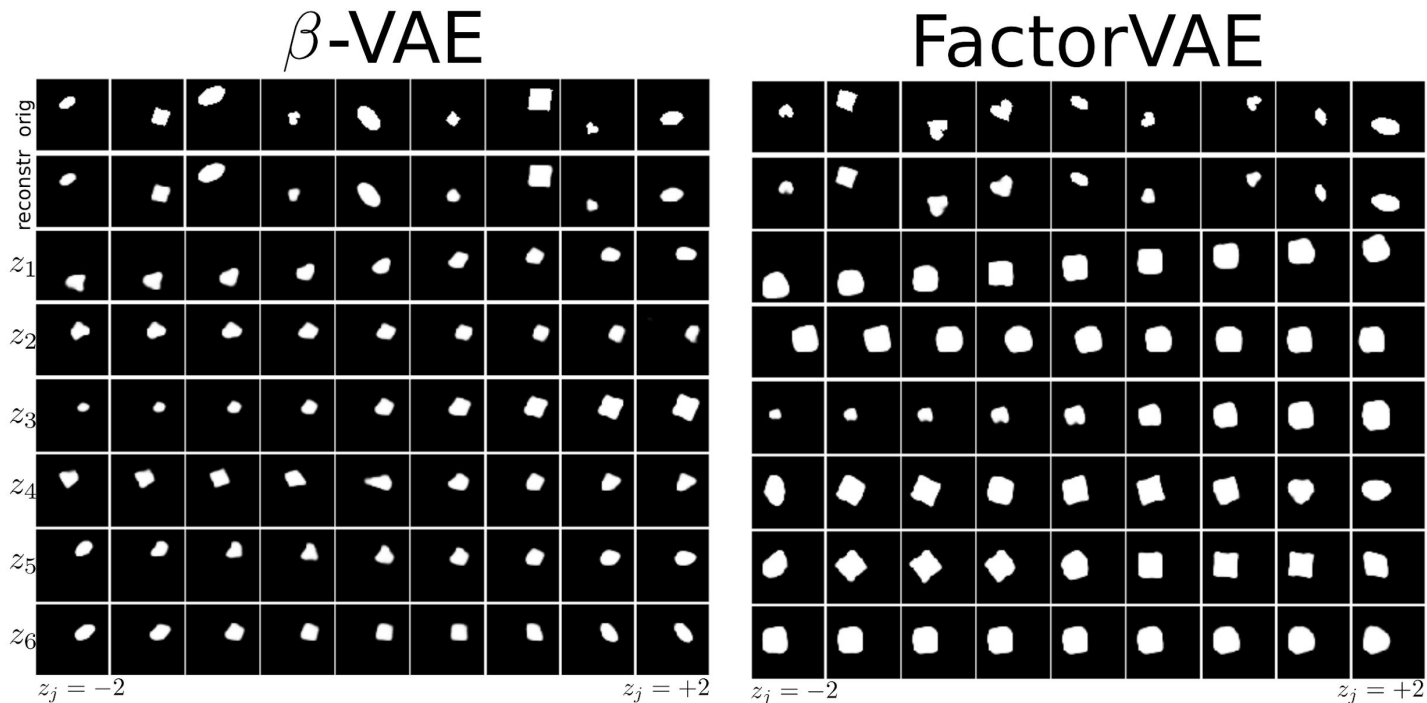
# FactorVAE

$$\begin{aligned} L_{\text{FactorVAE}}(p, q) = & \mathbb{E}_{q(z|x)} [\log p(x|z)] \\ & - D_{KL} [q(z|x) || p(z)] \\ & - \gamma D_{KL} (q(z) || \prod_j q(z_j)) \end{aligned}$$

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$z_i$  correlates with  $z_j$  if and only if  $i = j$

# $\beta$ -VAE and FactorVAE

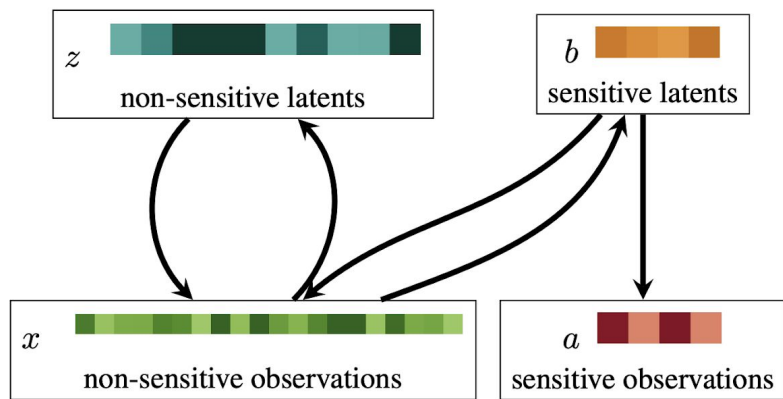


Models find x-position, y-position, and scale, but struggle to disentangle orientation and shape

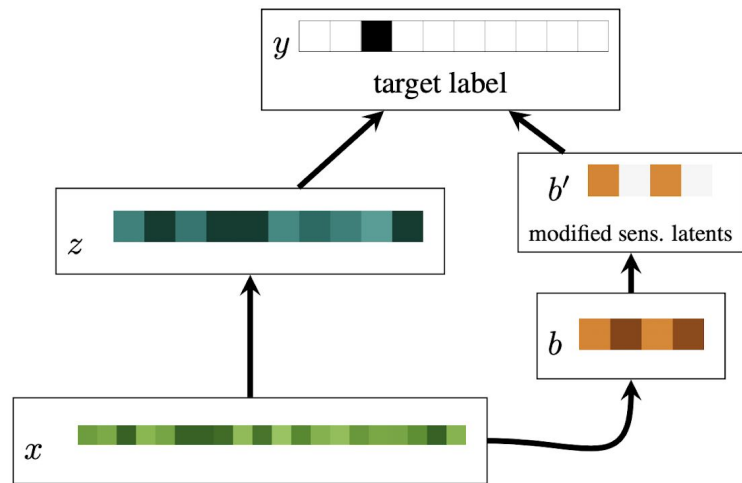
# Outline

- Disentangled Representations
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# Flexibly Fair Representation



Training



Testing

# Disentangled Fair Representations

$$q(z, b) = q(z) \prod_j q(b_j)$$

- Demographic Parity for Feature  $a_i$ 
  - Ignoring  $a_i$ , use instead  $[z, b] \setminus b_i$
  - or replace  $b_i$  with independent noise
- Compositional Procedure
  - use representation  $[z, b] \setminus \{b_i, b_j, b_k\}$  for fair combination  $\{a_i, a_j, a_k\}$

$z$  - non-sensitive dimension of the latent variables

$b$  - sensitive dimensions of the latent variables

# Flexibly Fair Representation

- $z \perp b_j \forall j$  (disentanglement of the non-sensitive and sensitive latent dimensions);
- $b_i \perp b_j \forall i \neq j$  (disentanglement of the various different sensitive dimensions);
- $\text{MI}(a_j, b_j)$  is large  $\forall j$  (predictiveness of each sensitive dimension);

# Flexibly Fair Representation

$$L_{\text{FFVAE}}(p, q) = \mathbb{E}_{q(z, b|x, a)} [\log p(x, a|z, b)]$$

$z \perp b_j$

$p(z, b) = p(z)p(b)$   
Standard Uniform  
Gaussian

$$- \underbrace{D_{KL} [q(z, b|x) || p(z, b)]}_{\beta\text{-VAE}}$$

$b_i \perp b_j \forall i \neq j$

$$- \underbrace{\gamma D_{KL} (q(z, b) || q(z) \prod_j q(b_j))}_{\text{factor-VAE}}$$

# Flexibly Fair Representation

$$\mathbb{E}_{q(z,b|x,a)} [\log p(x, a|z, b)] \xrightarrow{\quad} \mathbb{E}_{q(z,b|x)} [\log p(x|z, b) + \alpha \log p(a|b)]$$

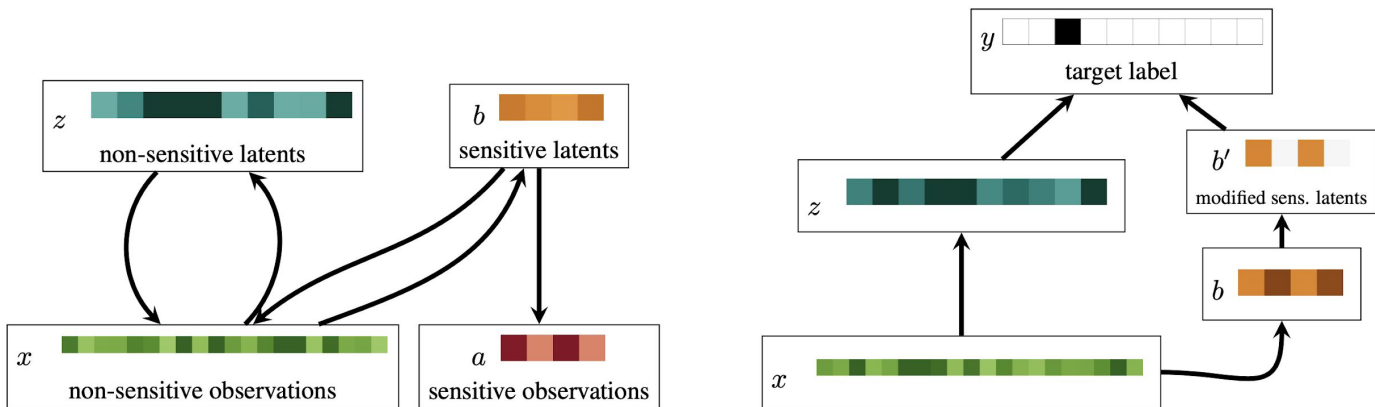
$p(x, a|z, b) = p(x|z, b)p(a|b)$

$$\begin{aligned} L_{\text{FFVAE}}(p, q) = & \mathbb{E}_{q(z,b|x)} [\log p(x|z, b) + \alpha \log p(a|b)] \\ & - \gamma D_{KL}(q(z, b) || q(z) \prod_j q(b_j)) \\ & - D_{KL}[q(z, b|x) || p(z, b)]. \end{aligned}$$



# Experiments

- Fair Classification
  - Make fair predictions
- Predictiveness
  - Train a classifier to predict sensitive attribute  $a_i$  from  $b_i$  alone
- Disentanglement
  - Train a classifier to predict sensitive attribute  $a_i$  from representations with  $b_i$  removed



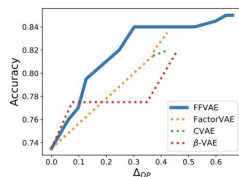
# Communities & Crime

## Fair Classification

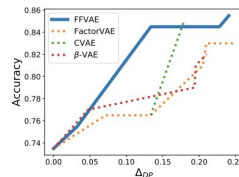
$$\Delta_{DP}(g) \triangleq d_g(\mathcal{Z}_0, \mathcal{Z}_1) = |\mathbb{E}_{\mathcal{Z}_0}[g] - \mathbb{E}_{\mathcal{Z}_1}[g]|$$

$$\Delta_{DP}(g) = 0 \iff g(Z) \perp A$$

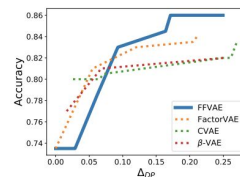
- Sensitive attributes:
  - racePctBlack (R)
  - blackPerCapIncome (B)
  - pctNotSpeakEnglWell (P)
- $y = \text{violentCrimesPerCapita}$



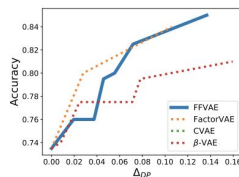
(a)  $a = R$



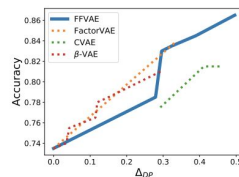
(b)  $a = B$



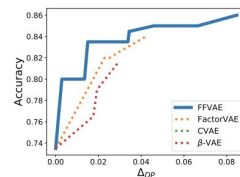
(c)  $a = P$



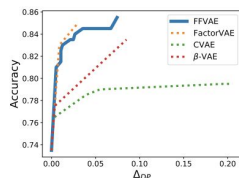
(d)  $a = R \vee B$



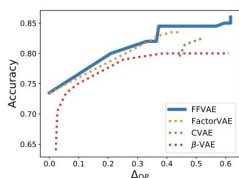
(e)  $a = R \vee P$



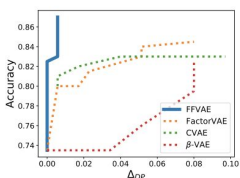
(f)  $a = B \vee P$



(g)  $a = R \wedge B$



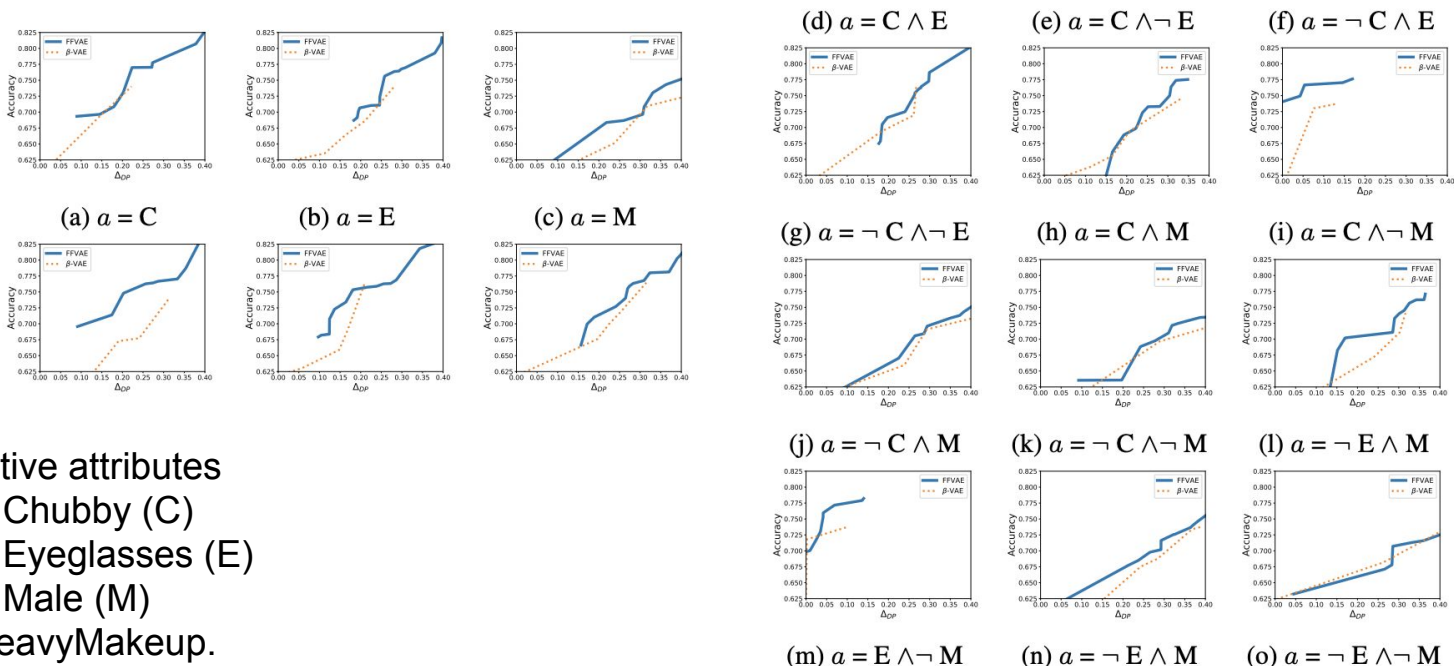
(h)  $a = R \wedge P$



(i)  $a = B \wedge P$

# CelebA

## Fair Classification

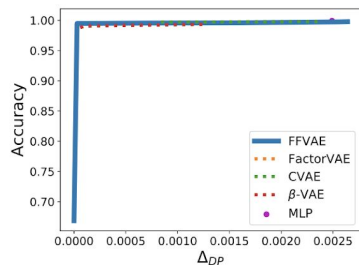


- Sensitive attributes
  - Chubby (C)
  - Eyeglasses (E)
  - Male (M)
- $y = \text{HeavyMakeup}$ .

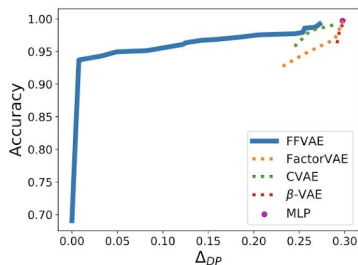
# DSpritesUnfair Dataset

## Fair Classification

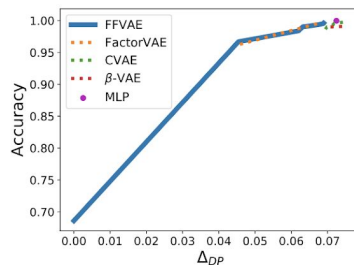
2D shapes procedurally generated from 6 ground truth independent latent factors. These factors are color, shape, scale, rotation, x and y positions of a sprite.



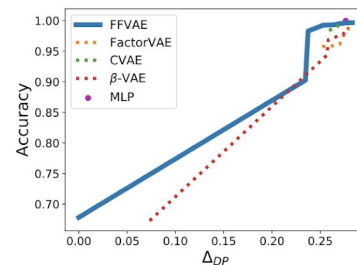
(a)  $a = \text{Scale}$



(b)  $a = \text{Shape}$



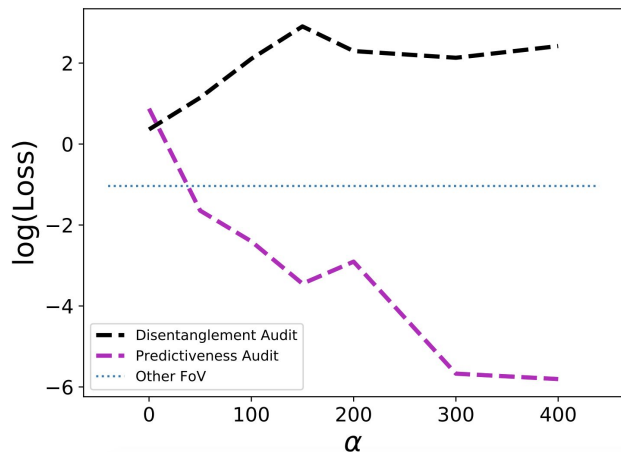
(c)  $a = \text{Shape} \wedge \text{Scale}$



(d)  $a = \text{Shape} \vee \text{Scale}$

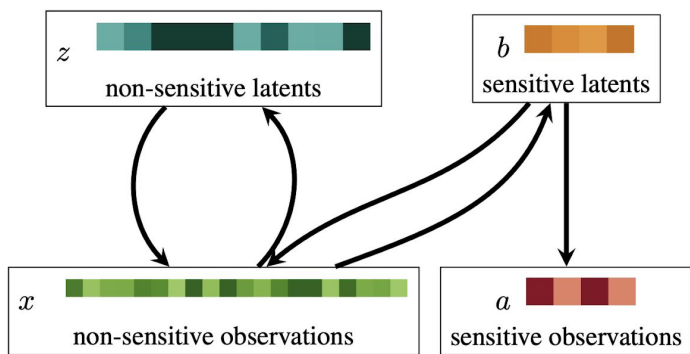
# DSpritesUnfair Dataset

- Disentanglement - Predict sensitive attribute  $a_i$  from  $b_i$  alone
- Predictiveness - Predict sensitive attribute  $a_i$  from representations with  $b_i$  removed

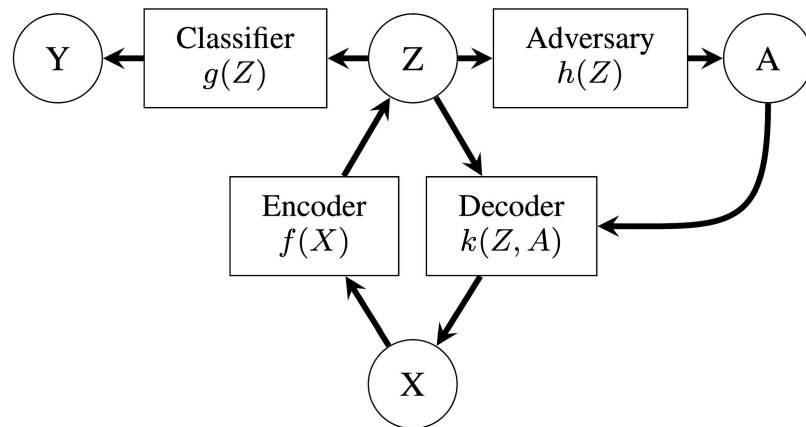


$$\begin{aligned} L_{\text{FFVAE}}(p, q) = & \mathbb{E}_{q(z, b|x)} [\log p(x|z, b) + \alpha \log p(a|b)] \\ & - \gamma D_{KL}(q(z, b) || q(z) \prod_j q(b_j)) \\ & - D_{KL}[q(z, b|x) || p(z, b)]. \end{aligned}$$

# Comparisons to Adversarial Learning



Flexibly Fair Representation



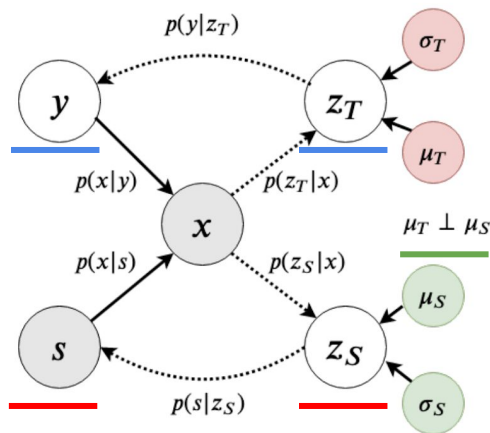
Adversarial Learning

# Outline

- Disentangled Representations
- Flexibly Fair Representation
- Orthogonal Disentangled Fair Representations
- Measurements for Disentangled Fair Representations

# Orthogonal Disentangled Fair Representations

- Train a fair representation that is
  - Disentangled
  - and Orthogonal





# Training Objective

$$\arg \min_{\theta_T, \theta_S, \phi_T, \phi_S} \mathcal{L}_T(\theta_T, \phi_T) + \mathcal{L}_S(\theta_S^*, \phi_S) + \lambda_E \mathcal{L}_E(\phi_S, \theta_T) + \lambda_{OD} \mathcal{L}_{OD}(\theta_T, \theta_S)$$

$$\mathcal{L}_T(\theta_T, \phi_T) = \text{KL}(p(\mathbf{y}|\mathbf{x}) \parallel q_{\phi_T}(\mathbf{y}|\mathbf{z}_T))$$

$$\mathcal{L}_S(\theta_S^*, \phi_S) = \text{KL}(p(\mathbf{s}|\mathbf{x}) \parallel q_{\phi_S}(\mathbf{s}|\mathbf{z}_S))$$

Matching the probabilities of ground truth (i.e.,  $\mathbf{y}$ ) and sensitive information (i.e.,  $\mathbf{s}$ ).

# Training Objective

$$\arg \min_{\theta_T, \theta_S, \phi_T, \phi_S} \mathcal{L}_T(\theta_T, \phi_T) + \mathcal{L}_S(\theta_S^*, \phi_S) + \lambda_E \mathcal{L}_E(\phi_S, \theta_T) + \lambda_{OD} \mathcal{L}_{OD}(\theta_T, \theta_S)$$

$$\mathcal{L}_E(\phi_S, \theta_T) = \text{KL}(q_{\phi_S}(\mathbf{s} | \mathbf{z}_T) \parallel \mathcal{U}(\mathbf{s}))$$

Makes sure that none sensitive information got leaked into the prediction  $\mathbf{z}_T$

# Training Objective

$$\arg \min_{\theta_T, \theta_S, \phi_T, \phi_S} \mathcal{L}_T(\theta_T, \phi_T) + \mathcal{L}_S(\theta_S^*, \phi_S) + \lambda_E \mathcal{L}_E(\phi_S, \theta_T) + \underline{\lambda_{OD} \mathcal{L}_{OD}(\theta_T, \theta_S)}$$

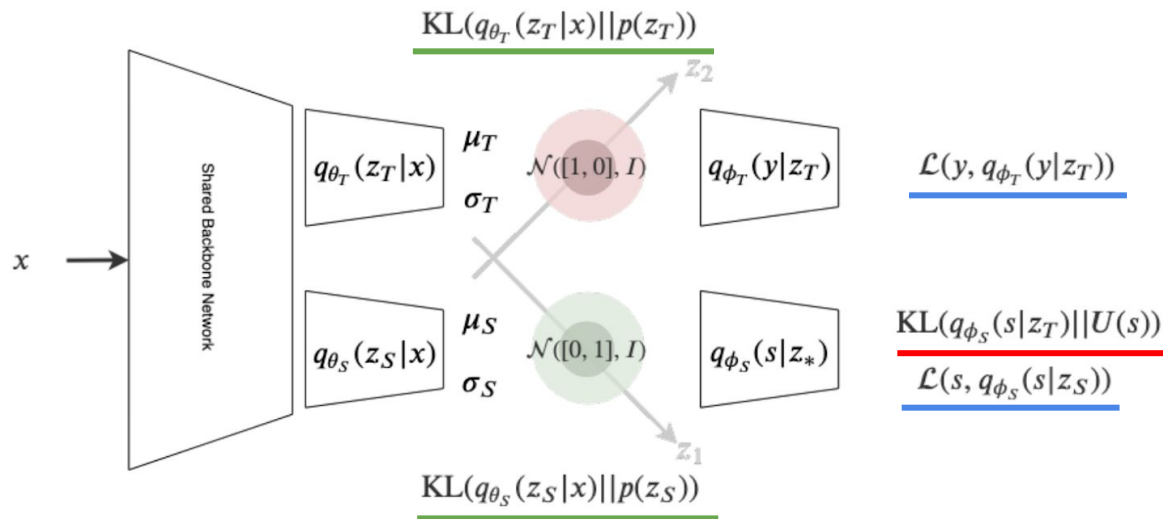
$$\mathcal{L}_{OD}(\theta_T, \theta_S) = \mathcal{L}_{z_T}(\theta_T) + \mathcal{L}_{z_S}(\theta_S)$$

$$\mathcal{L}_{z_T}(\theta_T) = \text{KL}(q_{\theta_T}(z_T | \mathbf{x}) \parallel p(z_T))$$

$$p(z_S) = \mathcal{N}([0, 1]^T, I) \quad p(z_T) = \mathcal{N}([1, 0]^T, I)$$

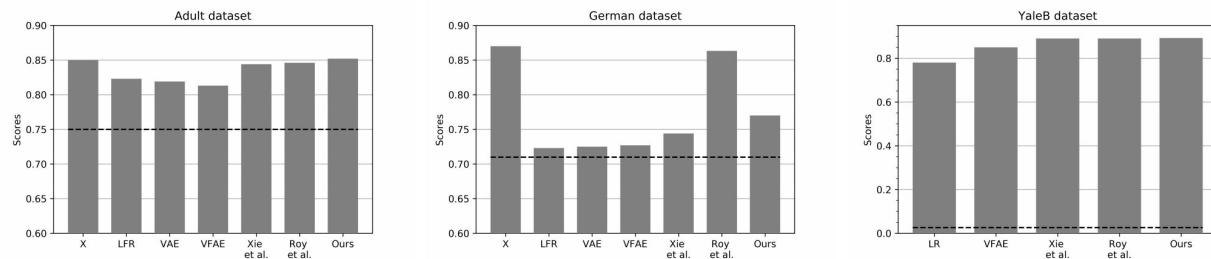
Enforces both Disentanglement and Orthogonality

# Training Objective

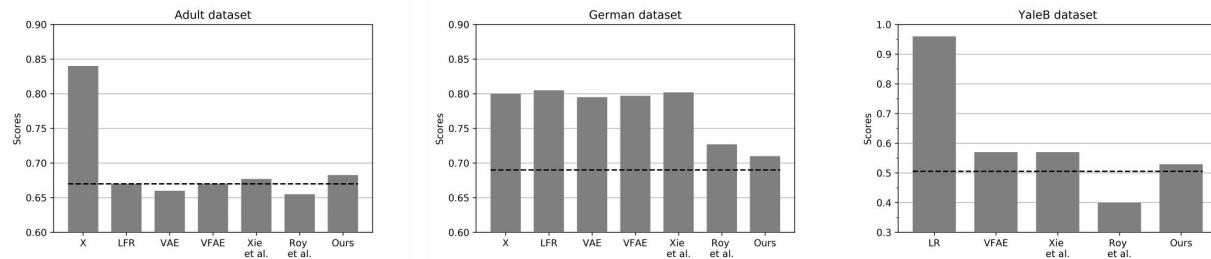


$$\arg \min_{\theta_T, \theta_S, \phi_T, \phi_S} \mathcal{L}_T(\theta_T, \phi_T) + \mathcal{L}_S(\theta_S^*, \phi_S) + \lambda_E \mathcal{L}_E(\phi_S, \theta_T) + \lambda_{OD} \mathcal{L}_{OD}(\theta_T, \theta_S)$$

# Adult, German, and extended YaleB



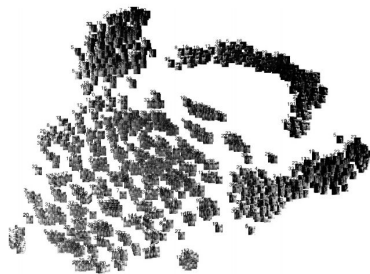
(a) Target attribute classification accuracy.



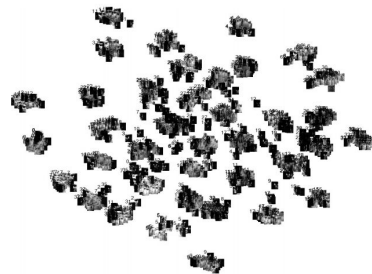
(b) Sensitive attribute classification accuracy.

# Visualizations on the Embeddings

YaleB faces



(a) t-SNE on  $x$



(b) t-SNE on  $z_T$



(c) t-SNE on  $z_S$

CIFAR 10



(d) t-SNE on  $x$



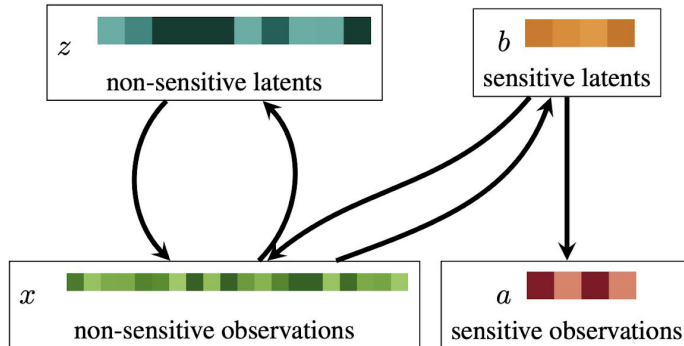
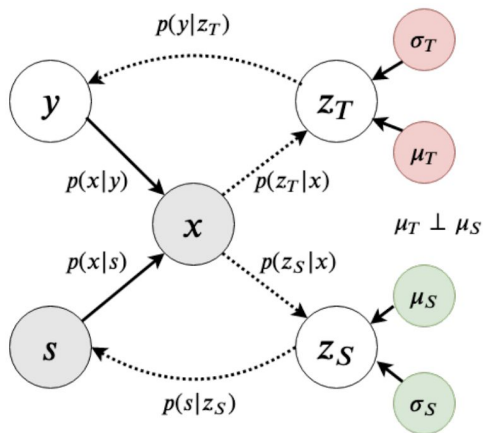
(e) t-SNE on  $z_T$



(f) t-SNE on  $z_S$

# Comparisons to Flexibly Fair Representation

- How do they handle leakage of sensitive information to the representations?
- How do they handle disentanglement?

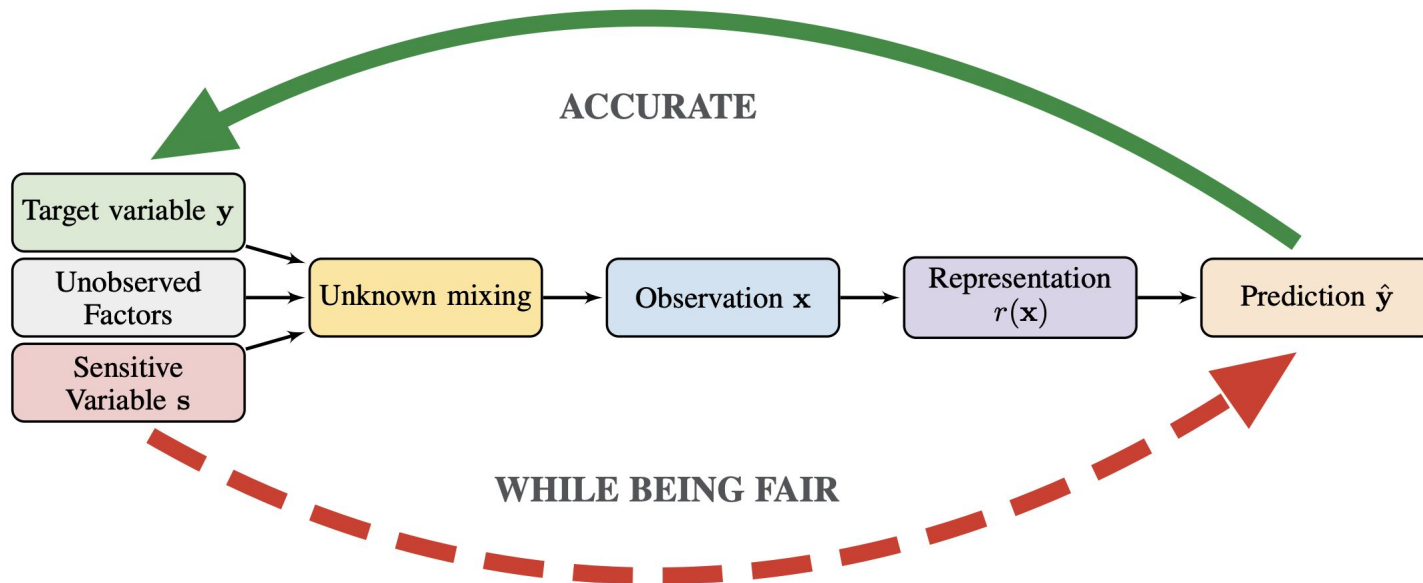


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# Measurements for Disentangled Fair Representations

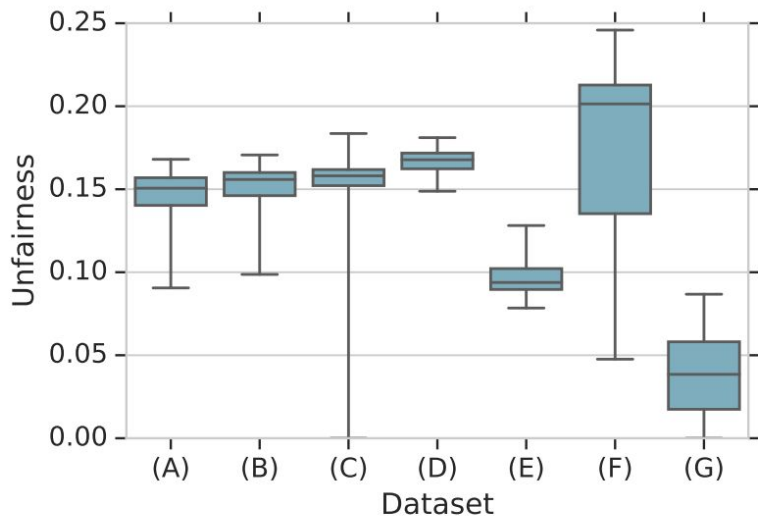


# Unfairness Measure

- Measuring Unfairness Without Ground Truth
  - Total Variation (TV) of prediction pairs across groups

$$\text{unfairness}(\hat{y}) = \frac{1}{|S|} \sum_s TV(p(\hat{y}), p(\hat{y} | \mathbf{s} = s)) \forall y$$

# Results Using Models Trained in [Locatello et al, 2019](#)



A - dSprites, B - Color-dSprites, C - Noisy-dSprites  
D - Scream-dSprites, E - SmallINORB, F - Cars3D, G - Shapes3D

BetaVAE Score	-69	-72	-20	-64	-57	-15	-50
FactorVAE Score	-69	-68	-41	-56	-55	-19	-40
MIG	-86	-81	-47	-65	-48	-32	-71
DCI Disentanglement	-92	-86	-71	-77	-55	-55	-95
Modularity	4	-6	28	4	41	-18	-55
SAP	-70	-64	-15	-60	-9	6	-53
	dSprites	Color-dSprites	Noisy-dSprites	Scream-dSprites	SmallINORB	Cars3D	Shapes3D

[Locatello et al, 2019](#)

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# Reading Assignments (Disentangled Fair Representations)

- Zhao, Han, Amanda Coston, Tameem Adel, and Geoffrey J. Gordon. Conditional learning of fair representations, ICLR 2020
- Zhao, Han, and Geoff Gordon. Inherent tradeoffs in learning fair representations, NeurIPS 2019
- He, Yuzi, Keith Burghardt, and Kristina Lerman. A Geometric Solution to Fair Representations, AAAI/ACM AI, Ethics, and Society 2020
- Ruoss, Anian, Mislav Balunović, Marc Fischer, and Martin Vechev. Learning Certified Individually Fair Representations, arXiv 2020
- Chiappa, Silvia, Ray Jiang, Tom Stepleton, Aldo Pacchiano, Heinrich Jiang, and John Aslanides. A general approach to fairness with optimal transport, AAAI 2020