CS448B :: 10 Nov 2011

Graph and Tree Layout

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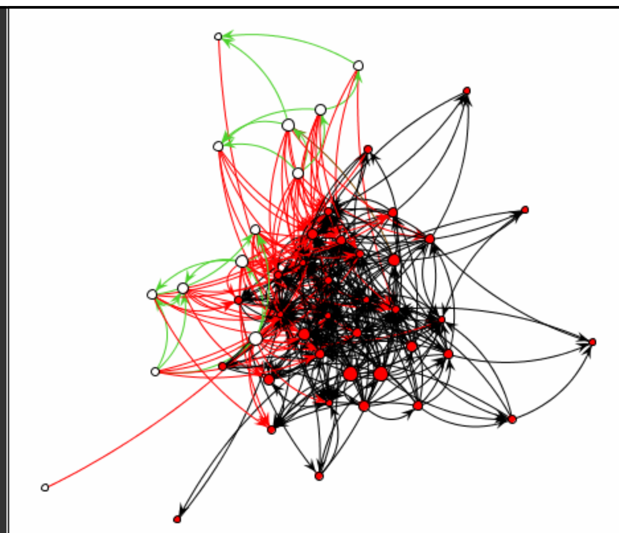
Topics

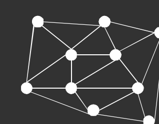
Graph and Tree Visualization
Tree Layout
Graph Layout

Goals
Overview of layout approaches and their strengths and weaknesses
Insight into implementation techniques



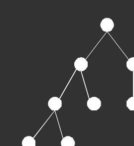
Graphs and Trees

Graphs
Model relations among data
Nodes and *edges*



Trees
Graphs with hierarchical structure
- Connected graph with N-1 edges

Nodes as *parents* and *children*



Spatial Layout

The primary concern of graph drawing is the spatial arrangement of nodes and edges

Often (but not always) the goal is to effectively depict the graph structure

- Connectivity, path-following
- Network distance
- Clustering
- Ordering (e.g., hierarchy level)

Applications of Tree / Graph Layout

Tournaments
Organization Charts
Genealogy
Diagramming (e.g., Visio)
Biological Interactions (Genes, Proteins)
Computer Networks
Social Networks
Simulation and Modeling
Integrated Circuit Design

Tree Layout

Tree Visualization

Indentation
- Linear list, indentation encodes depth

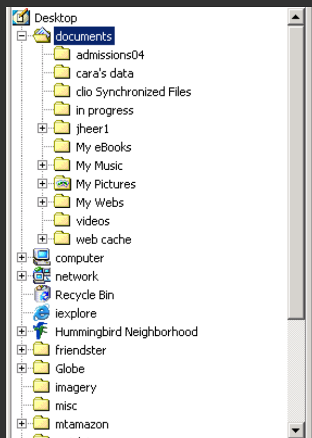
Node-Link diagrams
- Nodes connected by lines/curves

Enclosure diagrams
- Represent hierarchy by enclosure

Layering
- Relative position and alignment

Tree layout is fast: O(n) or O(n log n), enabling real-time layout for interaction.

Indentation



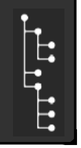
Places all items along vertically spaced rows

Indentation used to show parent/child relationships

Commonly used as a component in an interface

Breadth and depth contend for space

Often requires a great deal of scrolling

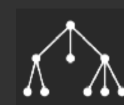


Node-Link Diagrams

Nodes are distributed in space, connected by straight or curved lines

Typical approach is to use 2D space to break apart breadth and depth

Often space is used to communicate hierarchical orientation (typically towards authority or generality)

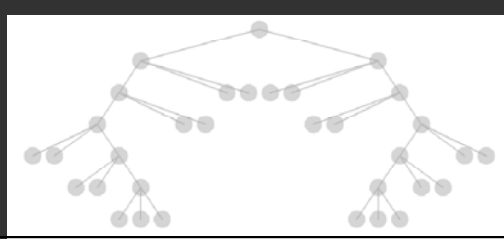


Basic Recursive Approach

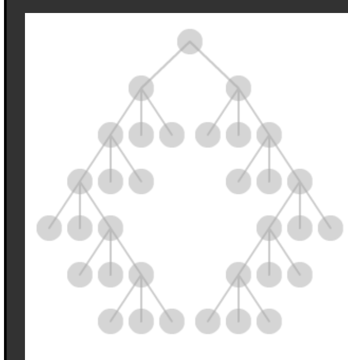
Repeatedly divide space for subtrees by leaf count

- Breadth of tree along one dimension
- Depth along the other dimension

Problem: exponential growth of breadth



Reingold & Tilford's Tidier Layout



Goal: make smarter use of space, maximize density and symmetry.

Originally for binary trees, extended by Walker to cover general case.

This was corrected by Buchheim et al to achieve a linear time algorithm.

Reingold-Tilford Layout

Design concerns

Clearly encode depth level

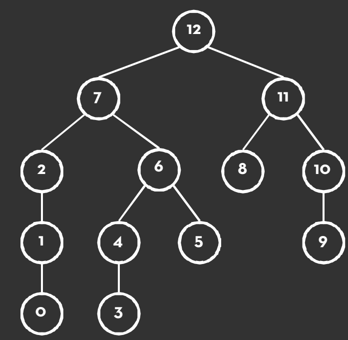
No edge crossings

Isomorphic subtrees drawn identically

Ordering and symmetry preserved

Compact layout (don't waste space)

Reingold-Tilford Algorithm



Reingold-Tilford Algorithm

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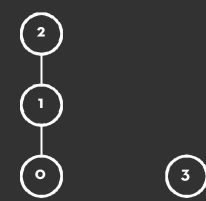

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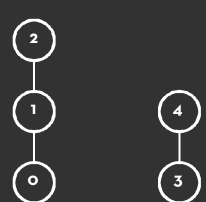

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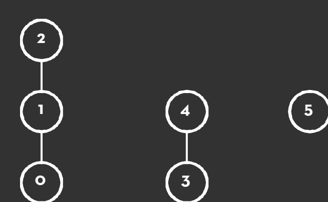

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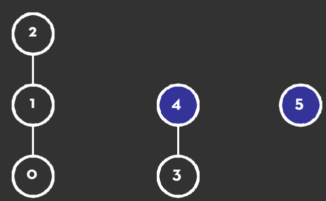

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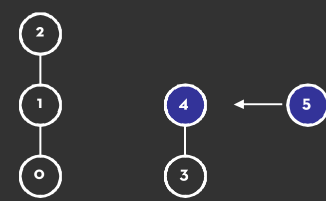

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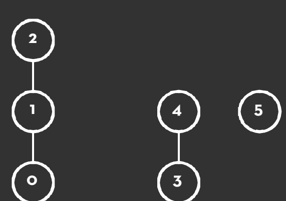

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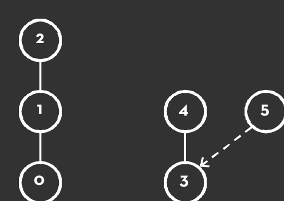

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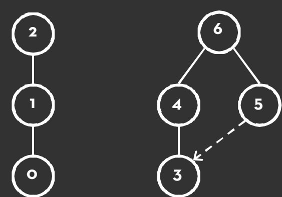

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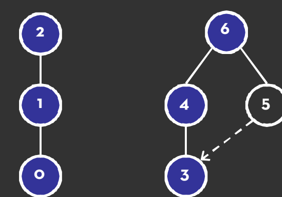

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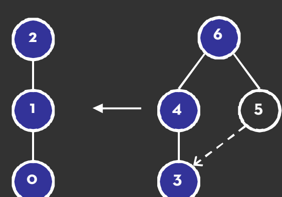

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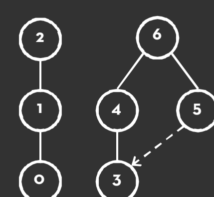

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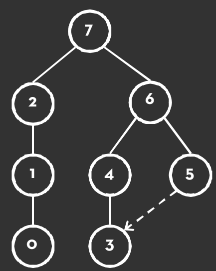

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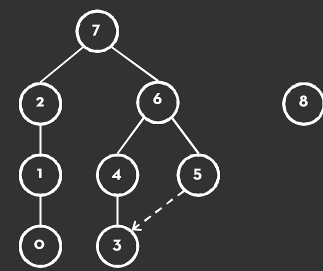

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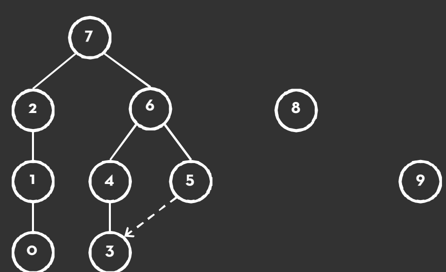

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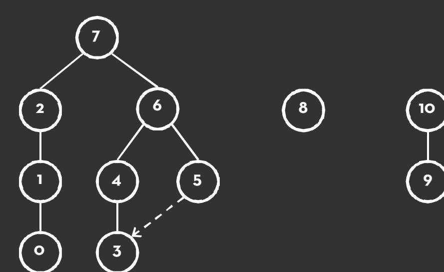

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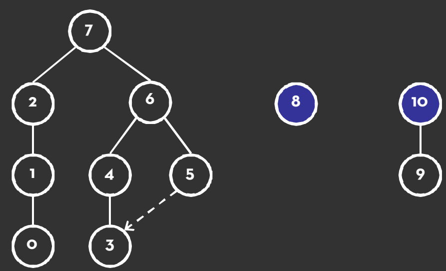

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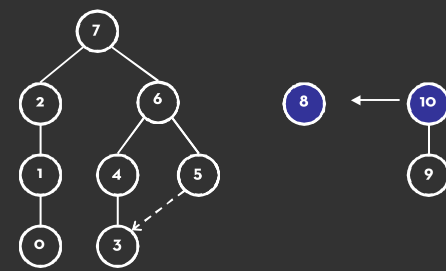

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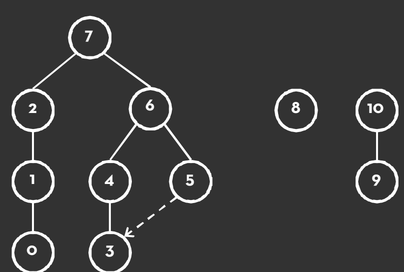

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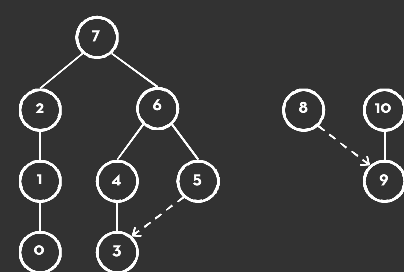

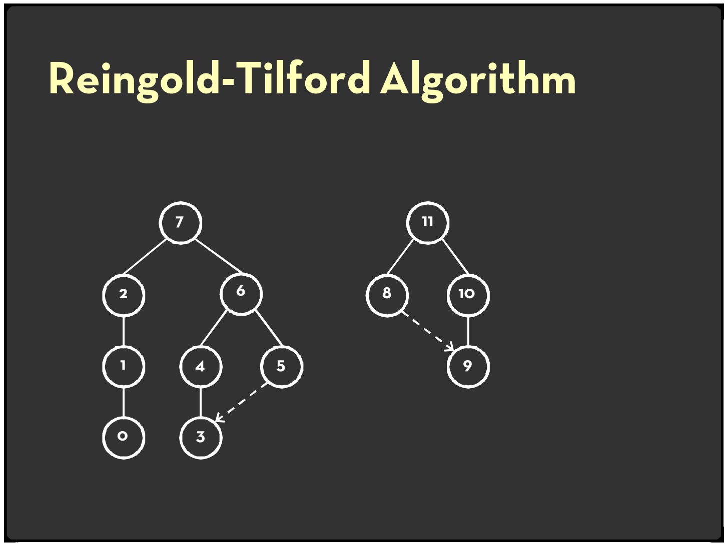
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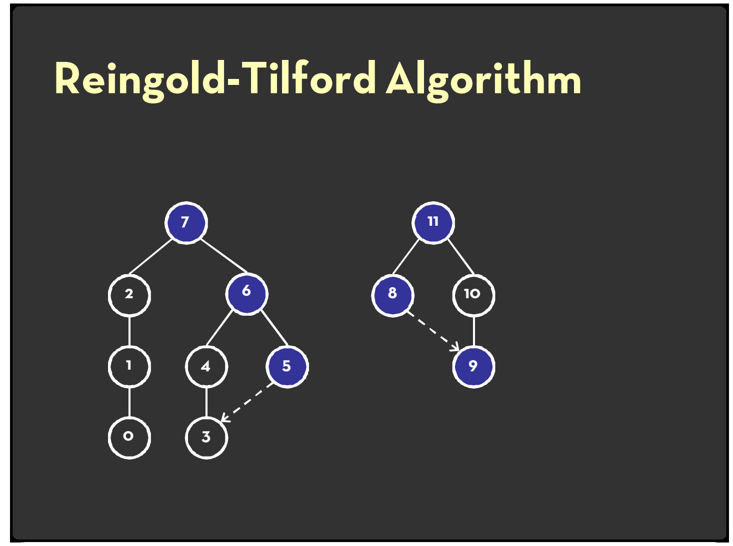
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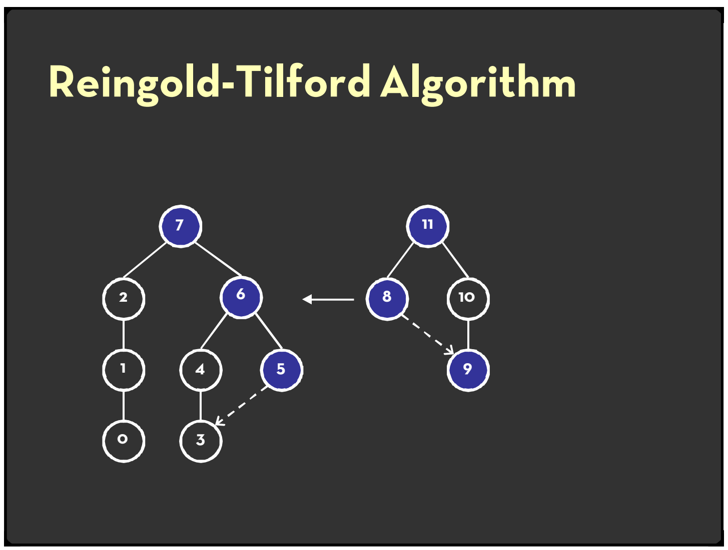
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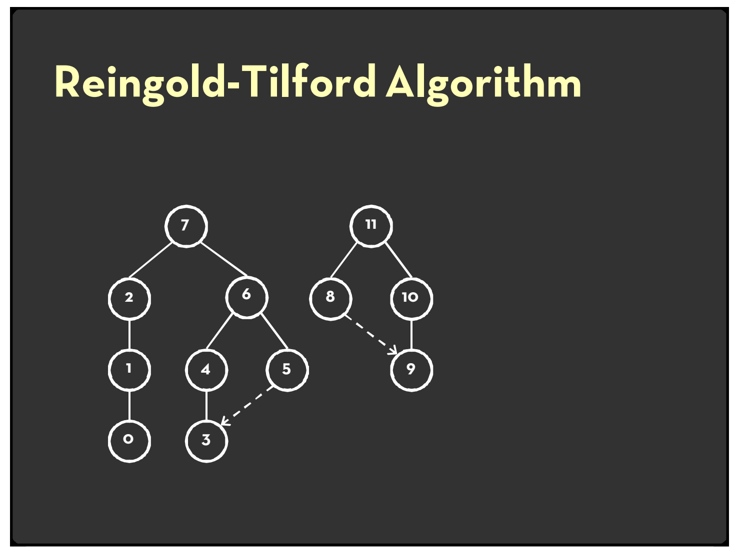
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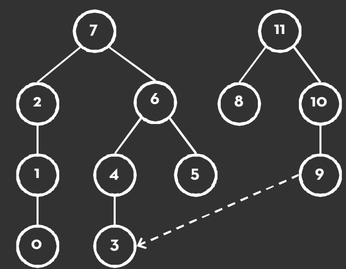

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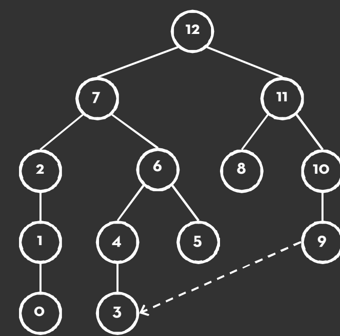
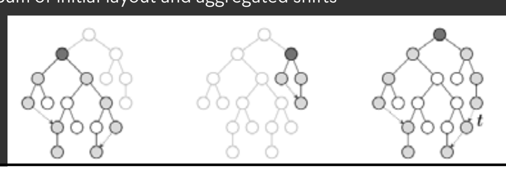
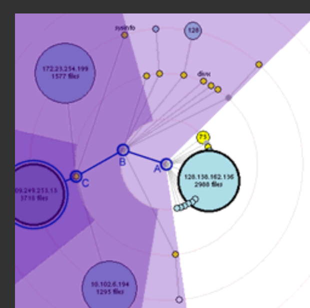


Reingold-Tilford Algorithm

- Linear algorithm – starts with bottom-up pass of the tree
- Y-coord by depth, arbitrary starting X-coord
- Merge left and right subtrees
  - Shift right as close as possible to left
    - Computed efficiently by maintaining subtree contours
  - "Shifts" in position saved for each node as visited
  - Parent nodes are centered above their children
- Top-down pass for assignment of final positions
  - Sum of initial layout and aggregated shifts



Radial Layout



Node-link diagram in polar co-ordinates.

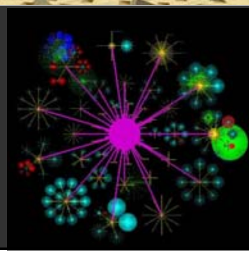
Radius encodes depth, with root in the center.

Angular sectors assigned to subtrees (typically uses recursive approach).

Reingold-Tilford approach can also be applied here.

Circular Drawing of Trees

Drawing in 3D to form **Cone Trees**



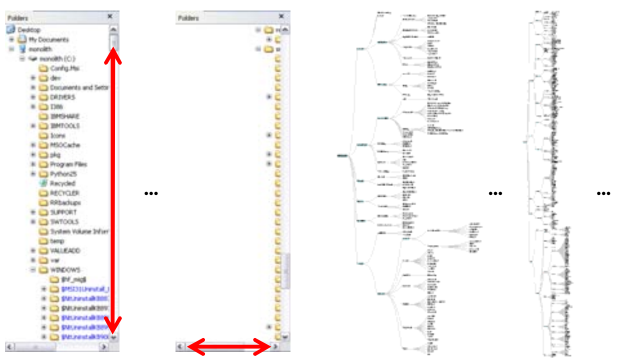
Balloon Trees can be described as a 2D variant of a Cone Tree. Not just a flattening process, as circles must not overlap.

Problems with Node-Link Diagrams

Scale
Tree breadth often grows exponentially
Even with tidier layout, quickly run out of space

Possible solutions
Filtering
Focus+Context
Scrolling or Panning
Zooming
Aggregation

Visualizing Large Hierarchies

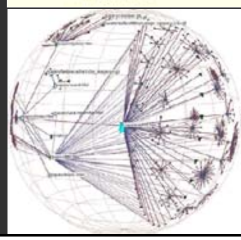


Indented Layout

Reingold-Tilford Layout

MC Escher, *Circle Limit IV*

Hyperbolic Layout

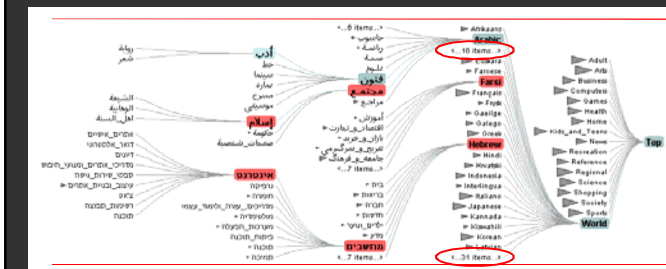


Perform tree layout in hyperbolic geometry, then project the result on to the Euclidean plane.

Why? Like tree breadth, the hyperbolic plane expands exponentially!

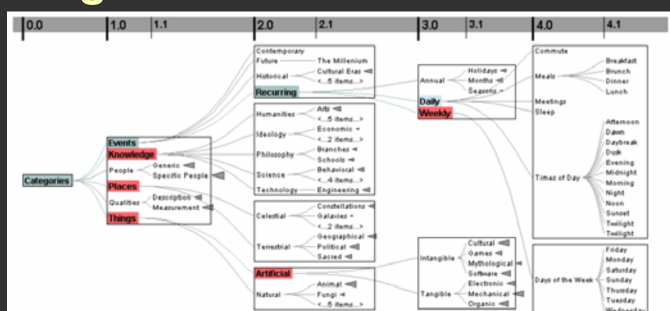
Also computable in 3D, projected into a sphere.

Degree-of-Interest Trees [AVI 04]



Space-constrained, multi-focal tree layout

Degree-of-Interest Trees



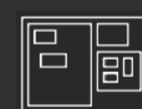
Cull “un-interesting” nodes on a per block basis until all blocks on a level fit within bounds.

Attempt to center child blocks beneath parents.

Enclosure Diagrams

Encode structure using spatial enclosure

Popularly known as **TreeMaps**



Benefits

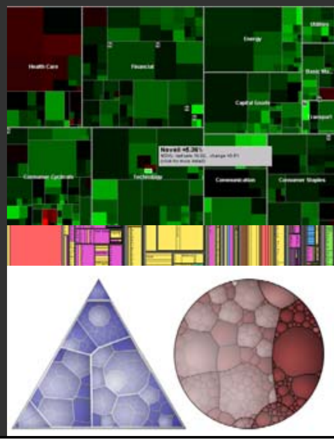
Provides a single view of an entire tree

Easier to spot large/small nodes

Problems

Difficult to accurately read depth

TreeMaps



Recursively fill space based on a size metric for nodes. Enclosure signifies hierarchy.

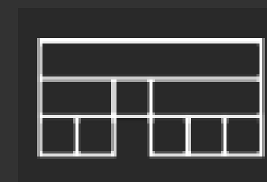
Additional measures can be taken to control aspect ratio of cells.

Often uses rectangles, but other shapes are possible, e.g., iterative Voronoi tesselation.

Layered Diagrams

Signify tree structure using

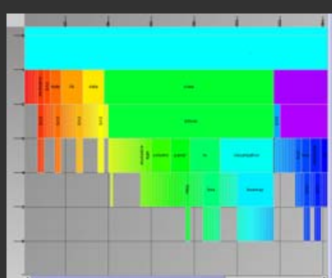
- Layering
- Adjacency
- Alignment



Involves recursive sub-division of space

We can apply the same set of approaches as in node-link layout.

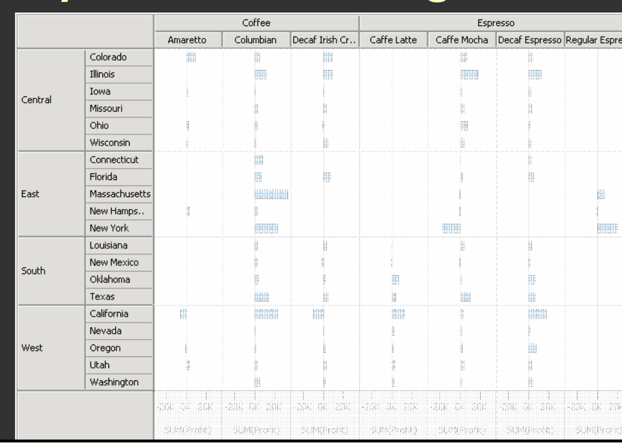
Icicle and Sunburst Trees



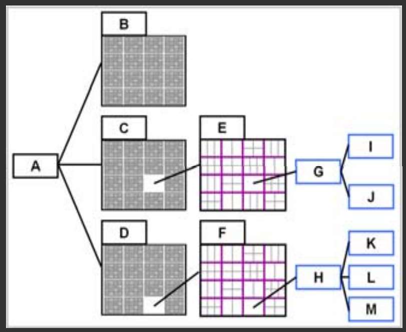
Higher-level nodes get a larger layer area, whether that is horizontal or angular extent.

Child levels are layered, constrained to parent's extent

Layered Tree Drawing



Hybrids are also possible...



"Elastic Hierarchies"

Node-link diagram with treemap nodes.

Administrivia

Final Project

Design a new visualization system or technique
Many options: new system, interaction technique, design study
6-8 page paper in conference paper format
2 Presentations: in-class report & final poster session

Schedule
Project Proposal: **Tuesday, Nov 15** *(end of day)*
In-Class Presentation: **Tuesday, Nov 29**
Poster Presentation: **Tuesday, Dec 13** *(5-7pm)*
Final Papers: **Thursday, Dec 15** *(5pm)*

Logistics
Groups of up to **4** people, graded individually
Clearly report responsibilities of each member

Final Project Ideas

Read the Final Project Wiki Page!
Also follow the links for suggested projects. A number of domain experts have provided project ideas and are excited to collaborate with you.

We **strongly** encourage you to consider working in a partnership with a domain expert, especially if you have difficulty formulating a problem-focused project idea.

Unsure? Come to office hours or schedule an appointment to discuss project ideas.

Final Project Proposal

Deliverables

Form project group (1-4 people)

Create project wiki page

Post project abstract (1-2 paragraphs)

- Should clearly state the problem, relevance & planned solution
- Start your related work search now to inform your proposal

Due Tues Nov 15 *(by end of day)*

Graph Layout

Approaches to Graph Drawing

Direct Calculation using Graph Structure

Tree layout on spanning tree

Hierarchical layout

Adjacency matrix layout

Optimization-based Layout

Constraint satisfaction

Force-directed layout

Attribute-Driven Layout

Layout using data attributes, not linkage

Spanning Tree Layout

Many graphs are tree-like or have useful spanning trees

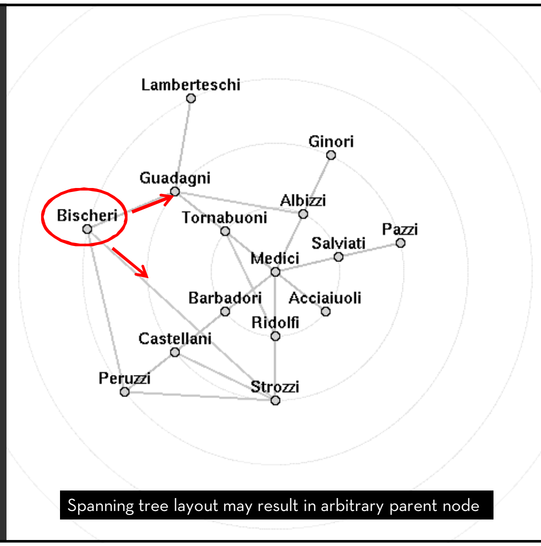
- Websites, Social Networks

Use tree layout on spanning tree of graph

- Trees created by BFS / DFS
- Min/max spanning trees

Fast tree layouts allow graph layouts to be recalculated at interactive rates

Heuristics may further improve layout

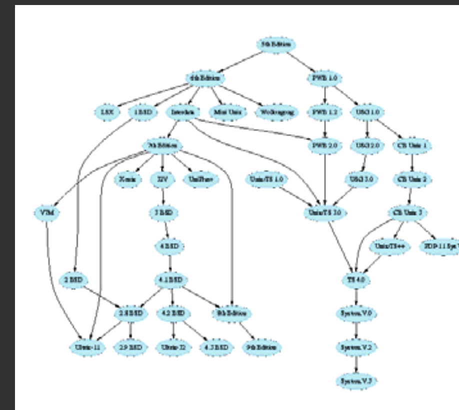


Spanning tree layout may result in arbitrary parent node

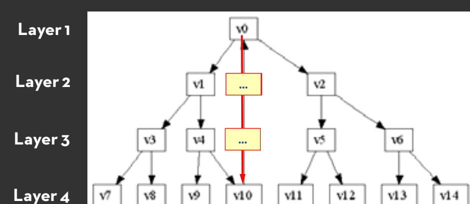
Sugiyama-style graph layout

Evolution of the UNIX operating system

Hierarchical layering based on descent

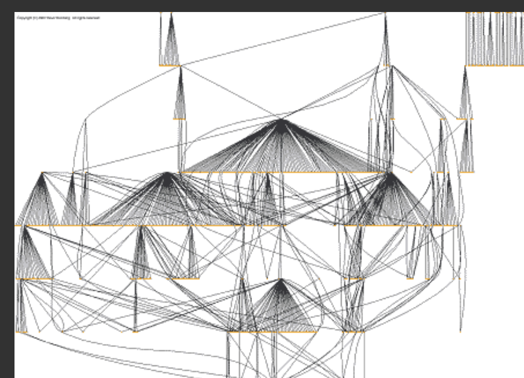


Sugiyama-style graph layout



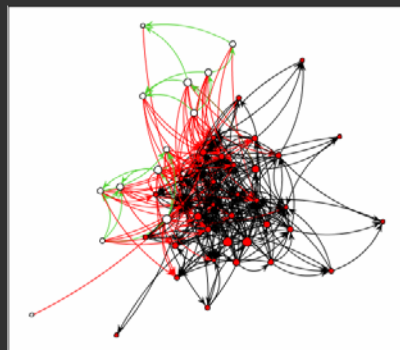
Reverse edges to remove cycles
Create dummy nodes to “fill in” missing layers
Assign nodes to hierarchy layers
Arrange nodes within layer, minimize edge crossings
Route edges – layout splines if needed

Hierarchical graph layout

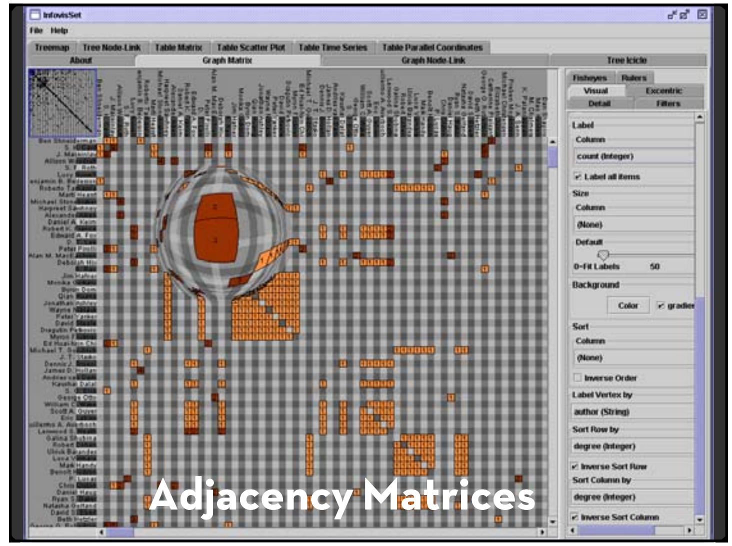


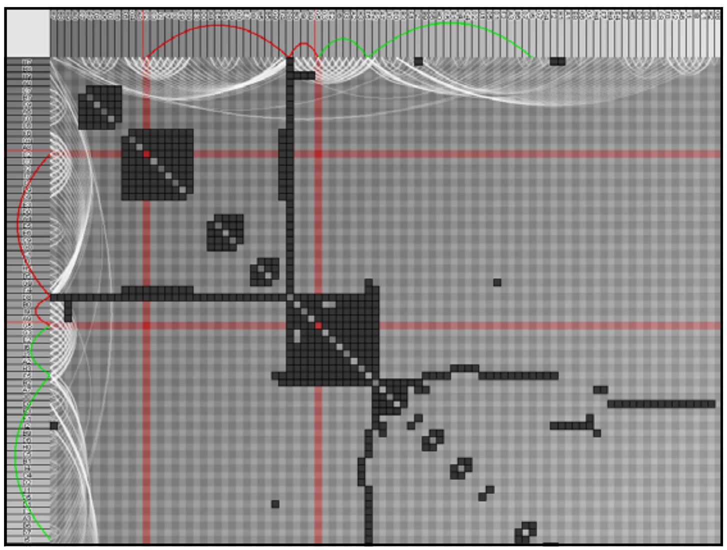
Gnutella network

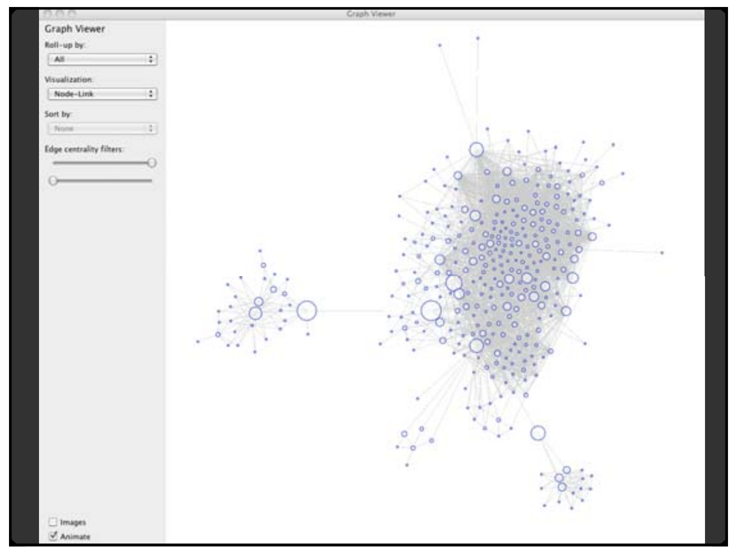
Limitations of Node-Link Layout

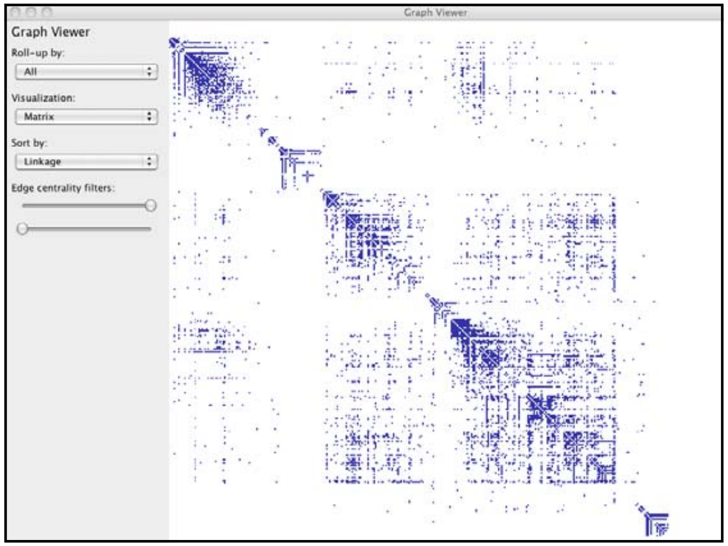


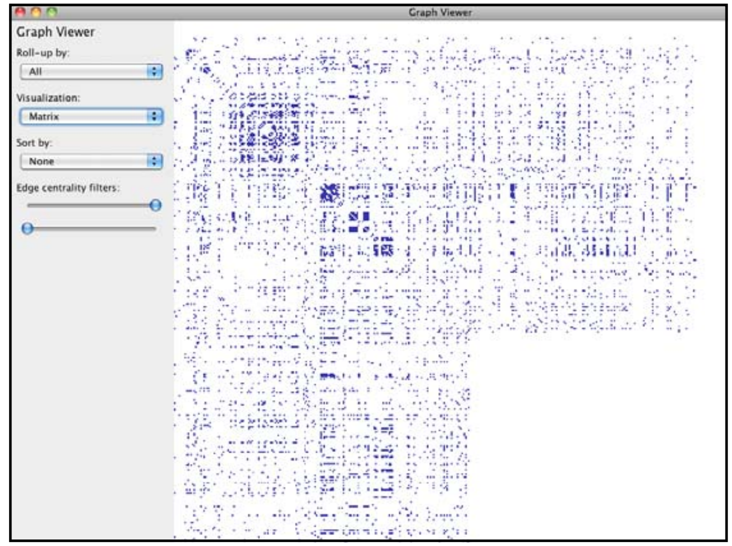
Edge-crossings and occlusion











Optimization Techniques

Treat layout as an *optimization problem*

- Define layout using an *energy model* and/or a set of *constraints*: equations the layout should try to obey
- Use optimization algorithms to solve

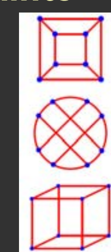
Regularly used for undirected graphs

- *Force-Directed Layout* most common

We can introduce directional constraints

- *DiG-CoLa* (Di-Graph Constrained Optimization Layout) [Dwyer 05]
- Iterative constraint relaxation

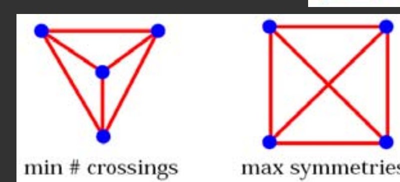
Optimizing "Aesthetic" Constraints



Minimize edge crossings
Minimize area
Minimize line bends
Minimize line slopes
Maximize smallest angle between edges
Maximize symmetry

but, can't do it all.

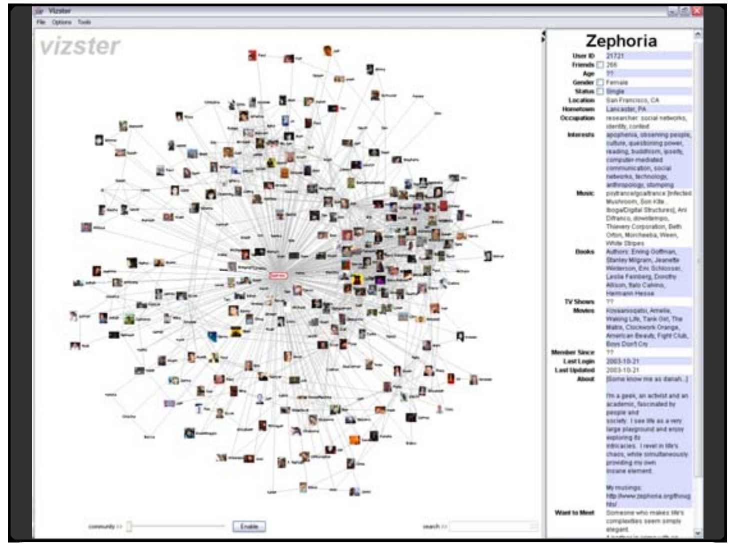
Optimizing these criteria is often NP-Hard, requiring approximations.



Force-Directed Layout

Nodes = charged particles $F = G{*}m_1{*}m_2 / (x_i - x_j)^2$
with air resistance $F = -b * v_i$
Edges = springs $F = -k * (x_i - x_j - L)$

Repeatedly calculate forces, update node positions
- Naïve approach $O(N^2)$
- Speed up to $O(N \log N)$ using quadtree or k-d tree
- Numerical integration of forces at each time step



Constrained Optimization Layout

Minimize stress function

$$\text{stress}(X) = \Sigma_{i<j}\, w_{ij}\, (\, \|X_i - X_j\| - d_{ij}\,)^2$$

- X: node positions, d: optimal edge length,
- w: normalization constants
- Use global (*majorization*) or localized (*gradient descent*) optimization

→ Says: Try to place nodes d_{ij} apart

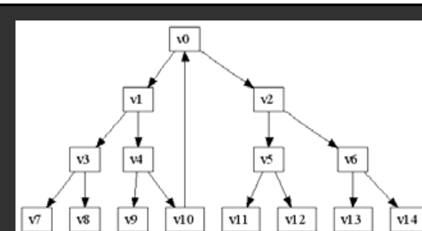
Add hierarchy ordering constraints

$$E_H(y) = \Sigma_{(i,j)\in E}\, (\, y_i - y_j - \delta_{ij}\,)^2$$

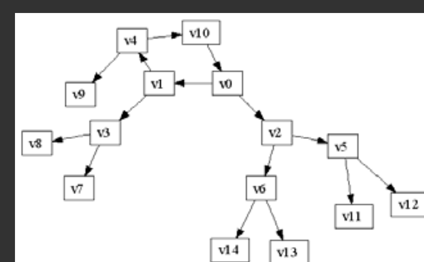
- y: node y-coordinates
- δ : edge direction (e.g., 1 for i→j, 0 for undirected)

→ Says: If *i* points to *j*, it should have a lower y-value

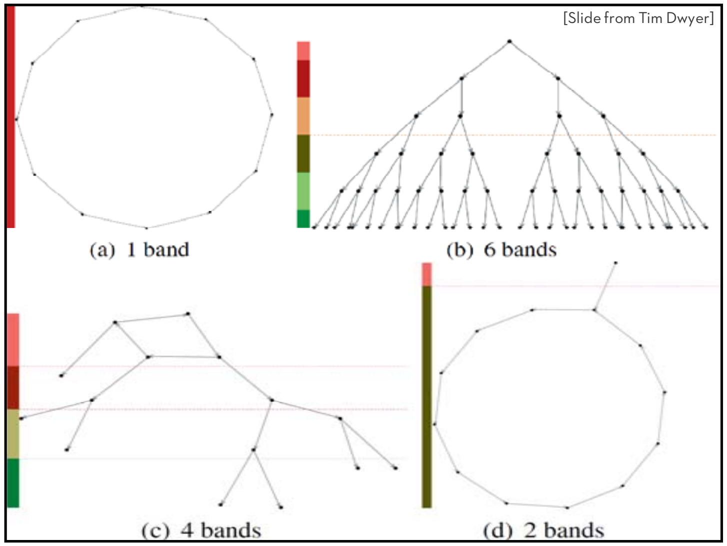
Sugiyama layout (dot)
Preserve tree structure



DiG-CoLa method
Preserve edge lengths



[Slide from Tim Dwyer]



Iterative Constraint Relaxation

Quadratic programming is complex to code and computationally costly. Is there a simpler way?

Iteratively relax each constraint [Dwyer 09]

- Given a constraint (e.g., $| x_i - x_j | = 5$)
- Simply push the nodes to satisfy

Each relaxation may clobber prior results

This typically (miraculously?) converges quickly and enables expressive constraints

Attribute-Driven Layout

Large node-link diagrams get messy!

Is there additional structure we can exploit?

Idea: Use data attributes to perform layout

- e.g., scatter plot based on node values

Dynamic queries and/or brushing can be used to explore connectivity

Attribute-Driven Layout

The “Skitter” Layout

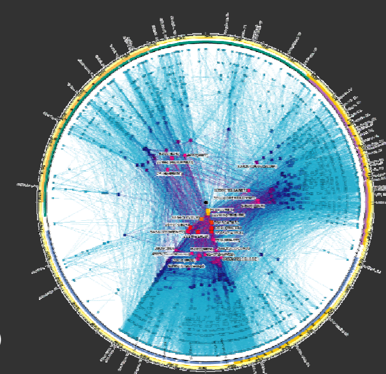
- Internet Connectivity
- Radial Scatterplot

Angle = Longitude

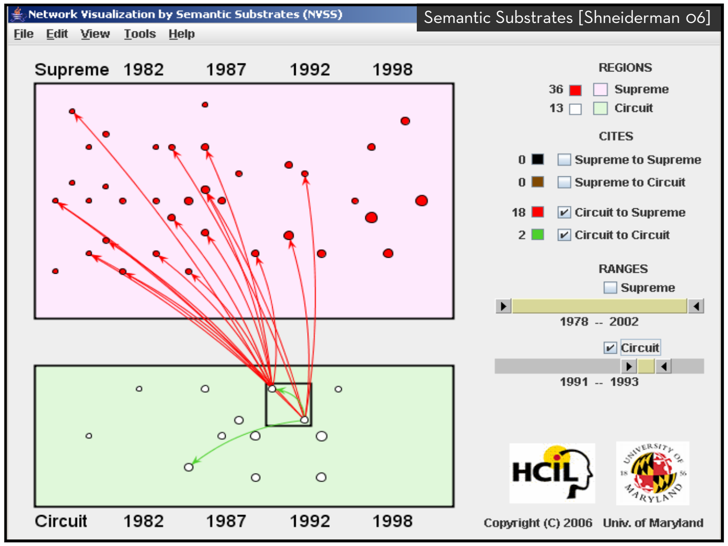
- Geography

Radius = Degree

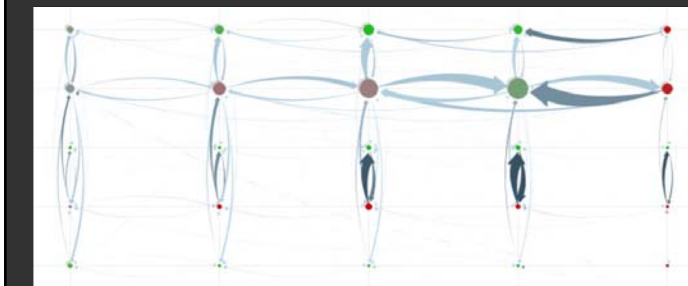
- # of connections
- (a statistic of the nodes)



Semantic Substrates [Shneiderman 06]

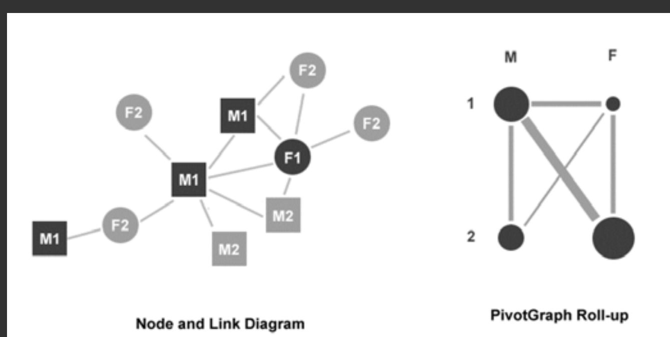


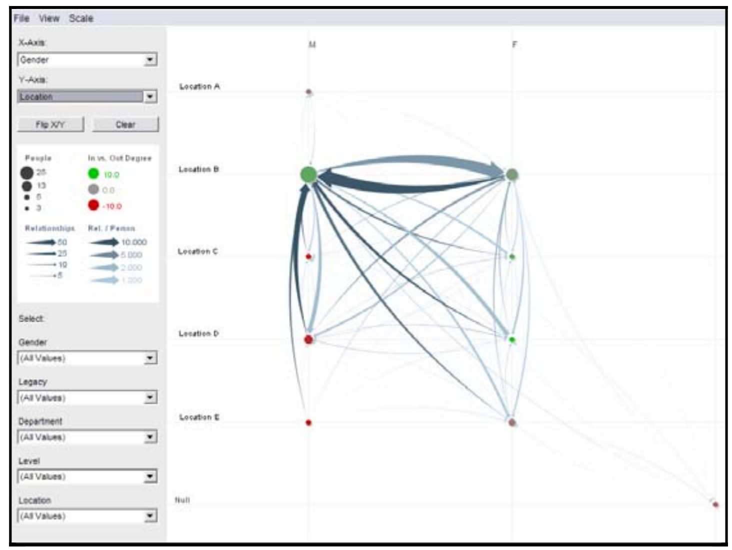
PivotGraph [Wattenberg 2006]



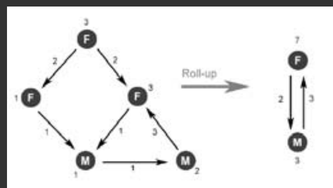
Layout aggregated graphs according to node attributes. Analogous to pivot tables and trellis display.

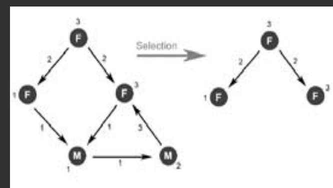
PivotGraph





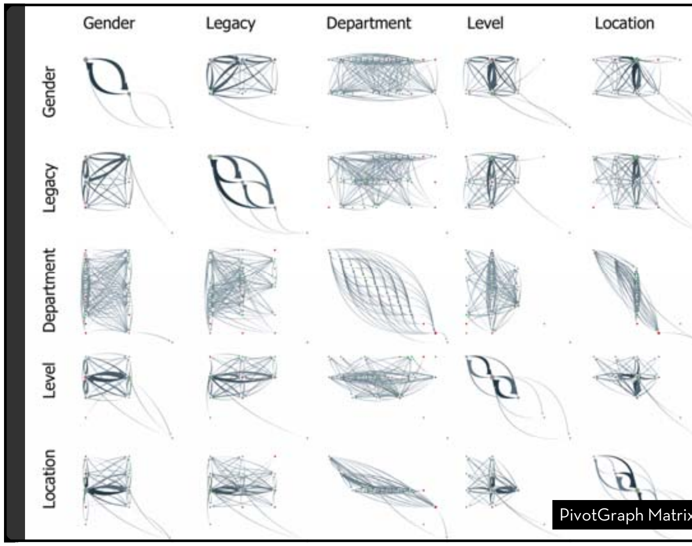
Operators





Roll-Up
Aggregate items with matching data values

Selection
Filter on data values

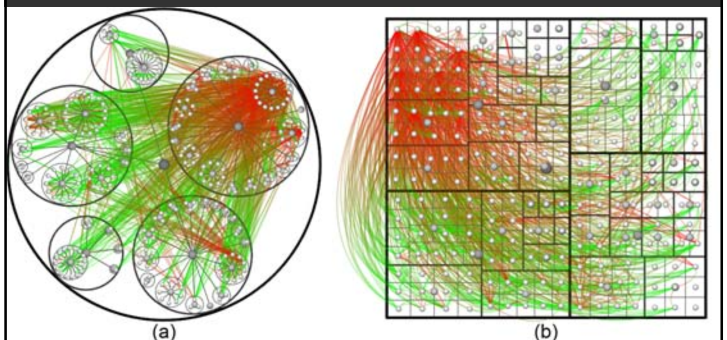


Limitations of PivotGraph

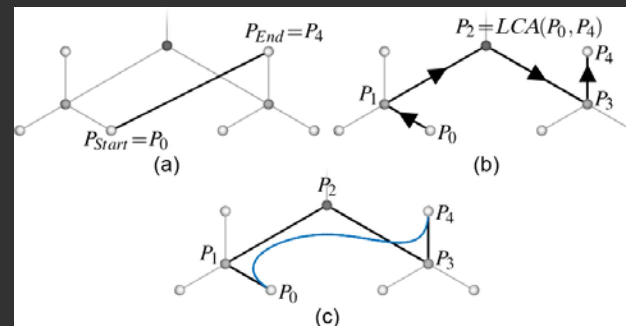
Only 2 variables (no nesting as in Tableau)
Doesn't support continuous variables
Multivariate edges?

Hierarchical Edge Bundling

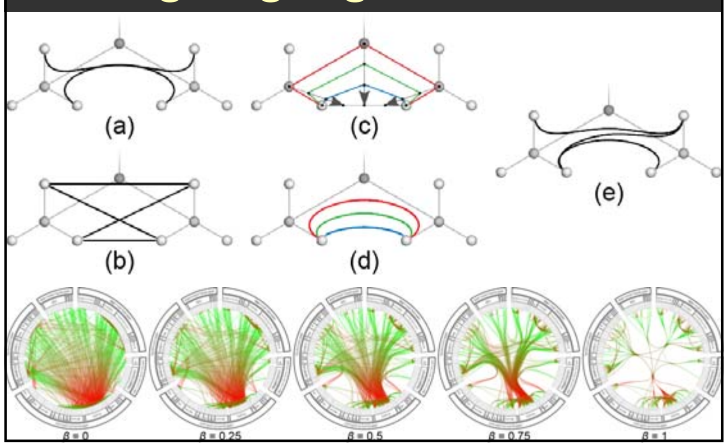
Trees with Adjacency Relations

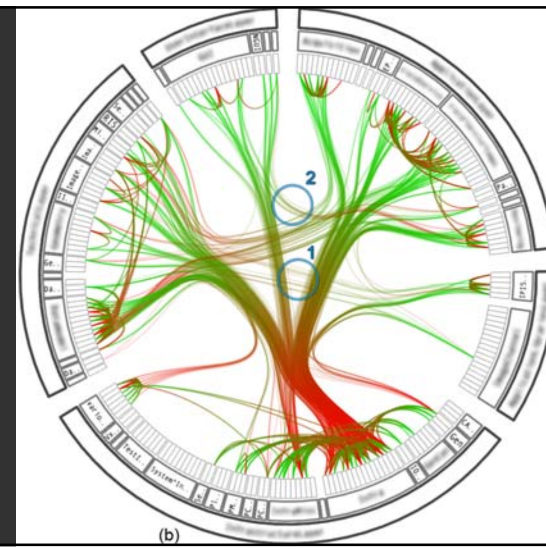


Bundle Edges along Hierarchy



Configuring Edge Tension





Summary

Tree Layout

Indented / Node-Link / Enclosure / Layers

How to address issues of scale?

- Filtering and Focus + Context techniques

Graph Layout

Tree layout over spanning tree

Hierarchical "Sugiyama" Layout

Optimization (Force-Directed Layout)

Attribute-Driven Layout